

ORF522 – Linear and Nonlinear Optimization

11. Interior-point methods implementation

Ed Forum

- For the interior-point methods, is the solution only "approximately correct" since it finds a solution within some tolerance of satisfying the optimality conditions? In this case, do we say that interior-point methods solve LPs in polynomial time (i.e. the worst-case complexity), or is there some other variant that can theoretically obtain an exact solution in polynomial time?
- How do people come up with this Logarithmic function? How to interpret the value of this Logarithmic function as the barriers?
- Are the ϵ 's just some values very close to 0 that the residual norms and sTy have to be less than or equal to? I assume since we distinguish them based on the primal, dual and gap that they're typically different values?
- Since σ 's bounds are $[0,1]$ inclusive, does that mean there are cases where you would take a full Newton or centering step?

Recap

(Sparse) Cholesky factorization

Every positive definite matrix A can be factored as

$$A = PLL^T P^T \longrightarrow P^T AP = LL^T$$

P permutation, L lower triangular

Permutations

- Reorder rows/cols of A with P to (heuristically) get **sparser** L
- P depends only on sparsity pattern of A (unlike LU factorization)
- If A is dense, we can set $P = I$

Cost

- If A dense, typically $O(n^3)$ but usually much less
- It depends on the number of nonzeros in A , sparsity pattern, etc.
- Typically 50% faster than LU (need to find only one matrix)

Optimality conditions

	Primal	Dual
minimize $c^T x$	minimize $c^T x$	maximize $-b^T y$
subject to $Ax \leq b$	subject to $Ax + s = b$ $s \geq 0$	subject to $A^T y + c = 0$ $y \geq 0$

Optimality conditions

$$Ax + s - b = 0$$

$$A^T y + c = 0$$

$$s_i y_i = 0$$

$$s, y \geq 0$$

Central path

$$\begin{aligned} &\text{minimize} && c^T x - \tau \sum_{i=1}^m \log(s_i) \\ &\text{subject to} && Ax + s = b \end{aligned}$$

Set of points $(x^*(\tau), s^*(\tau), y^*(\tau))$
with $\tau > 0$ such that

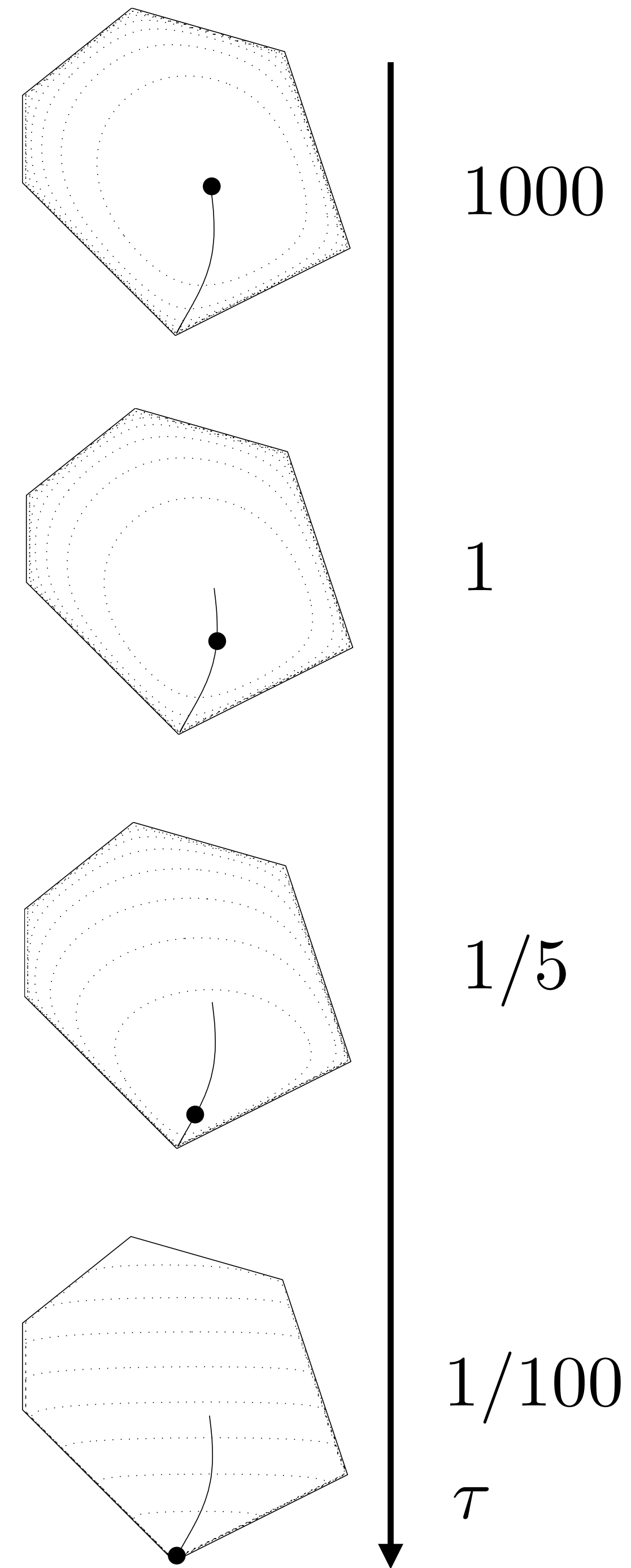
$$Ax + s - b = 0$$

$$A^T y + c = 0$$

$$s_i y_i = \tau$$

$$s, y \geq 0$$

**Analytic
Center**
 $\tau \rightarrow \infty$



Main idea

Follow central path as $\tau \rightarrow 0$

Strict complementarity

Primal

$$\begin{aligned} &\text{minimize} && c^T x \\ &\text{subject to} && Ax + s = b \\ &&& s \geq 0 \end{aligned}$$

Dual

$$\begin{aligned} &\text{maximize} && -b^T y \\ &\text{subject to} && A^T y + c = 0 \\ &&& y \geq 0 \end{aligned}$$

Theorem

If the two problems have feasible solutions, then there exist feasible s and y with a **strict complementary sparsity** pattern:

$$y_i > 0, s_i = 0 \quad \text{or} \quad y_i = 0, s_i > 0$$

$$\text{In other words, } s_i + y_i > 0$$

Proof (left as exercise)

Details in [Theorem 10.6, LP]

Main idea

Optimality conditions

$$h(x, s, y) = \begin{bmatrix} Ax + s - b \\ A^T y + c \\ SY\mathbf{1} \end{bmatrix} = 0$$
$$s, y \geq 0$$

$$S = \mathbf{diag}(s)$$
$$Y = \mathbf{diag}(y)$$

- Apply variants of Newton's method to solve $h(x, s, y) = 0$
- Enforce $s, y > 0$ (strictly) at every iteration
- **Motivation** avoid getting stuck in “corners”

Algorithm step

Linear system

$$\begin{bmatrix} 0 & A & I \\ A^T & 0 & 0 \\ S & 0 & Y \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta x \\ \Delta s \end{bmatrix} = \begin{bmatrix} -r_p \\ -r_d \\ -SY\mathbf{1} + \sigma\mu\mathbf{1} \end{bmatrix}$$

Duality measure

$$\mu = \frac{s^T y}{m}$$

Centering parameter

$$\sigma \in [0, 1]$$

$\sigma = 0 \Rightarrow$ Newton step

$\sigma = 1 \Rightarrow$ Centering step towards $(x^*(\mu), s^*(\mu), y^*(\mu))$

Line search to enforce $s, y > 0$

$$(x, s, y) \leftarrow (x, s, y) + \alpha(\Delta x, \Delta s, \Delta y)$$

Primal-dual path-following algorithm

Initialization

1. Given (x_0, s_0, y_0) such that $s_0, y_0 > 0$

Iterations

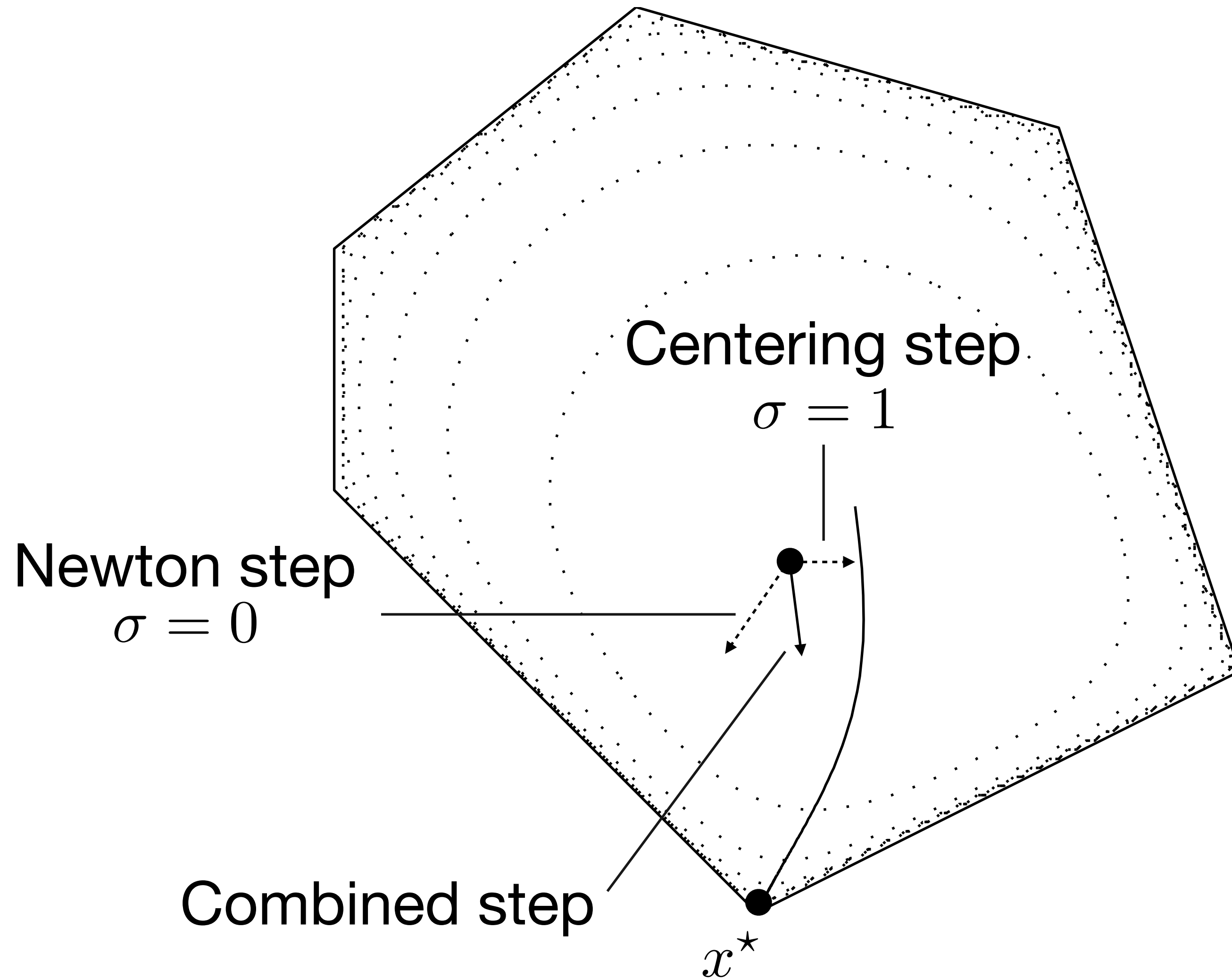
1. Choose $\sigma \in [0, 1]$

2. Solve
$$\begin{bmatrix} 0 & A & I \\ A^T & 0 & 0 \\ S & 0 & Y \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta x \\ \Delta s \end{bmatrix} = \begin{bmatrix} -r_p \\ -r_d \\ -SY + \sigma\mu\mathbf{1} \end{bmatrix} \text{ where } \mu = s^T y / m$$

3. Find maximum α such that $y + \alpha\Delta y > 0$ and $s + \alpha\Delta s > 0$

4. Update $(x, s, y) \leftarrow (x, s, y) + \alpha(\Delta x, \Delta s, \Delta y)$

Path-following algorithm idea



Centering step

It brings towards the **central path** and is usually biased towards $s, y > 0$.
No progress on duality measure μ

Newton step

It brings towards the **zero duality measure** μ . Quickly violates $s, y > 0$.

Combined step

Best of both worlds with longer steps

Today's lecture

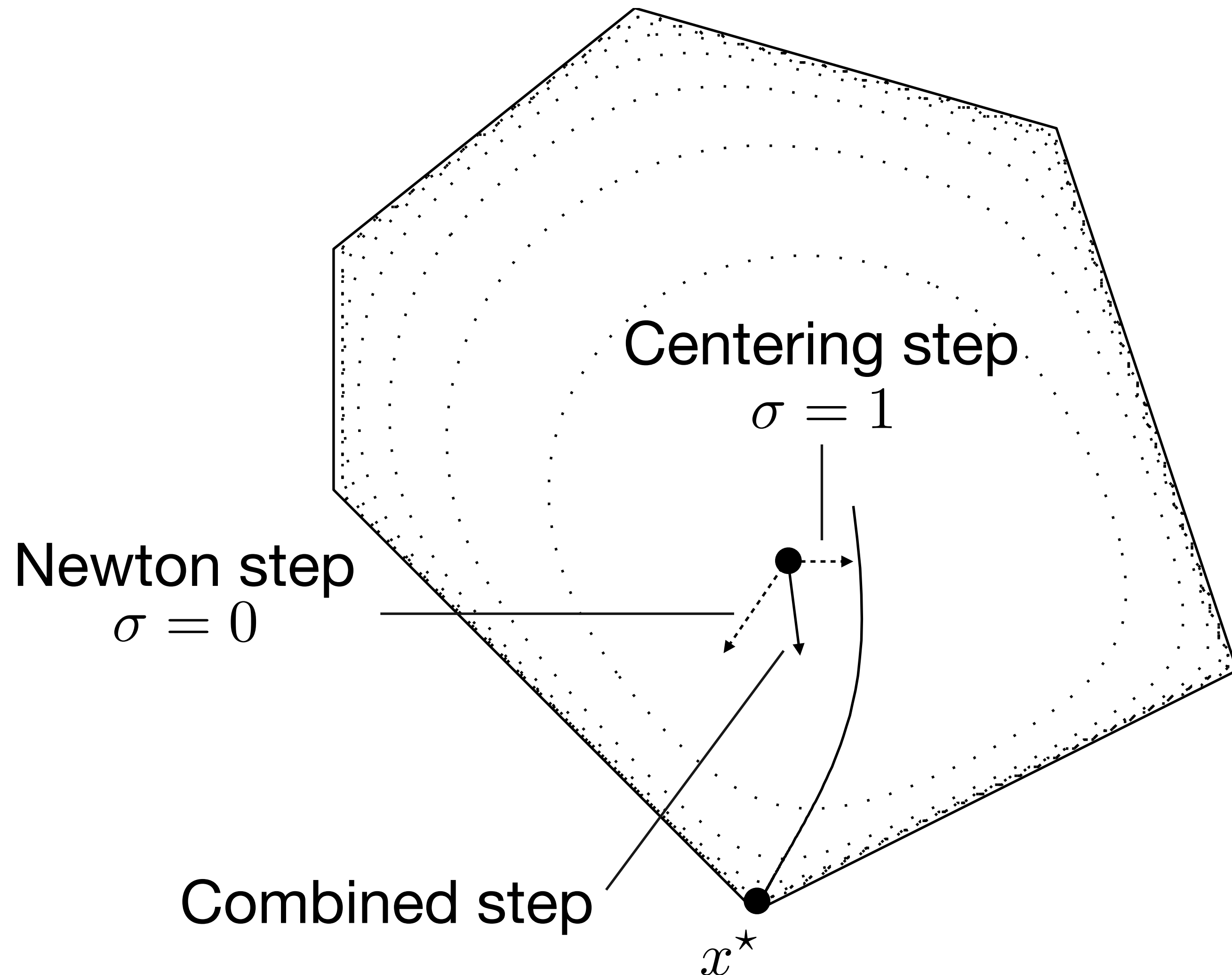
[Chapter 14, NO][Chapter 22, LP]

- Mehrotra predictor-corrector algorithm
- Implementation details
- Homogeneous self-dual embedding
- Interior-point vs simplex

Predictor-corrector algorithm

Main idea:

Predict and select centering parameter



Predict

Compute Newton direction

Estimate

How good is the Newton step?
(how much can μ decrease?)

Select centering parameter

Very roughly:

Pick $\sigma \approx 0$ if Newton step is good

Pick $\sigma \approx 1$ if Newton step is bad

Select centering parameter

Newton step

$$(\Delta x_a, \Delta s_a, \Delta y_a)$$

Maximum step-size

$$\alpha_p = \max\{\alpha \in [0, 1] \mid s + \alpha \Delta s_a \geq 0\}$$

$$\alpha_d = \max\{\alpha \in [0, 1] \mid y + \alpha \Delta y_a \geq 0\}$$

Duality measure candidate
(after Newton step)

$$\mu_a = \frac{(s + \alpha_p \Delta s_a)^T (y + \alpha_d \Delta y_a)}{m}$$

Centering parameter heuristic σ

$$\sigma = \left(\frac{\mu_a}{\mu} \right)^3$$

Mehrotra correction

Newton step

$$\begin{bmatrix} 0 & A & I \\ A^T & 0 & 0 \\ S & 0 & Y \end{bmatrix} \begin{bmatrix} \Delta y_a \\ \Delta x_a \\ \Delta s_a \end{bmatrix} = \begin{bmatrix} -r_p \\ -r_d \\ -SY\mathbf{1} \end{bmatrix} \longrightarrow s_i(\Delta y_a)_i + y_i(\Delta s_a)_i + s_i y_i = 0$$

Full step

$$(s_i + (\Delta s_a)_i)(y_i + (\Delta y_a)_i) = (\Delta s_a)_i(\Delta y_a)_i \neq 0$$

Complementarity violation depends on step length

Corrected direction

$$\begin{bmatrix} 0 & A & I \\ A^T & 0 & 0 \\ S & 0 & Y \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta x \\ \Delta s \end{bmatrix} = \begin{bmatrix} -r_p \\ -r_d \\ -SY\mathbf{1} - \Delta S_a \Delta Y_a \mathbf{1} + \sigma \mu \mathbf{1} \end{bmatrix}$$

$$\Delta S_a = \text{diag}(\Delta s_a)$$

$$\Delta Y_a = \text{diag}(\Delta y_a)$$

Mehrotra predictor-corrector algorithm

Initialization

Given (x, s, y) such that $s, y > 0$

1. Termination conditions

$$r_p = Ax + s - b, \quad r_d = A^T y + c, \quad \mu = (s^T y)/m$$

If $\|r_p\|, \|r_d\|, \mu$ are small, **break** Optimal solution (x^*, s^*, y^*)

2. Newton step (affine scaling)

$$\begin{bmatrix} 0 & A & I \\ A^T & 0 & 0 \\ S & 0 & Y \end{bmatrix} \begin{bmatrix} \Delta y_a \\ \Delta x_a \\ \Delta s_a \end{bmatrix} = \begin{bmatrix} -r_p \\ -r_d \\ -SY\mathbf{1} \end{bmatrix}$$

Mehrotra predictor-corrector algorithm

3. Barrier parameter

$$\alpha_p = \max\{\alpha \in [0, 1] \mid s + \alpha\Delta s_a \geq 0\}$$

$$\alpha_d = \max\{\alpha \in [0, 1] \mid y + \alpha\Delta y_a \geq 0\}$$

$$\mu_a = \frac{(s + \alpha_p\Delta s_a)^T (y + \alpha_d\Delta y_a)}{m}$$

$$\sigma = \left(\frac{\mu_a}{\mu}\right)^3$$

4. Corrected direction

$$\begin{bmatrix} 0 & A & I \\ A^T & 0 & 0 \\ S & 0 & Y \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta x \\ \Delta s \end{bmatrix} = \begin{bmatrix} -r_p \\ -r_d \\ -SY\mathbf{1} - \Delta S_a\Delta Y_a\mathbf{1} + \sigma\mu\mathbf{1} \end{bmatrix}$$

Mehrotra predictor-corrector algorithm

5. Update iterates

$$\alpha_p = \max\{\alpha \geq 0 \mid s + \alpha\Delta s \geq 0\}$$

$$\alpha_d = \max\{\alpha \geq 0 \mid y + \alpha\Delta y \geq 0\}$$

$$(x, s) = (x, s) + \min\{1, \eta\alpha_p\}(\Delta x, \Delta s)$$

$$y = y + \min\{1, \eta\alpha_d\}\Delta y$$

Avoid corners

$$\eta = 1 - \epsilon \approx 0.99$$

Implementation details

Search equations

Step 2 (**Newton**) and 4 (**Corrected direction**) solve equations of the form

$$\begin{bmatrix} 0 & A & I \\ A^T & 0 & 0 \\ S & 0 & Y \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta x \\ \Delta s \end{bmatrix} = \begin{bmatrix} b_y \\ b_x \\ b_s \end{bmatrix}$$

The **Newton** step right hand side:

$$\begin{bmatrix} b_y \\ b_x \\ b_s \end{bmatrix} = \begin{bmatrix} -r_p \\ -r_d \\ -SY\mathbf{1} \end{bmatrix}$$

The **corrector** step right hand side:

$$\begin{bmatrix} b_y \\ b_x \\ b_s \end{bmatrix} = \begin{bmatrix} -r_p \\ -r_d \\ -SY\mathbf{1} - \Delta S_a \Delta Y_a \mathbf{1} + \sigma \mu \mathbf{1} \end{bmatrix}$$

Solving the search equations

Our linear system is not symmetric

$$\begin{bmatrix} 0 & A & I \\ A^T & 0 & 0 \\ S & 0 & Y \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta x \\ \Delta s \end{bmatrix} = \begin{bmatrix} b_y \\ b_x \\ b_s \end{bmatrix}$$

Substitute last equation, $\Delta s = Y^{-1}(b_s - S\Delta y)$, into first

$$\begin{bmatrix} -Y^{-1}S & A \\ A^T & 0 \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta x \end{bmatrix} = \begin{bmatrix} b_y - Y^{-1}b_s \\ b_x \end{bmatrix}$$

Substitute first equation, $\Delta y = S^{-1}Y(A\Delta x - b_y + Y^{-1}b_s)$, into second

$$A^T S^{-1}Y A \Delta x = b_x + A^T S^{-1}Y b_y - A^T S^{-1}b_s$$

Simplified linear system

Coefficient matrix

$$B = A^T S^{-1} Y A$$

Characteristics

- A is **large** and **sparse**
- $S^{-1}Y$ is **positive** and **diagonal**, different at each iteration
- B is **positive definite** if $\text{rank}(A) = n$
- Sparsity pattern of B is the **pattern** of $A^T A$ (independent of $S^{-1}Y$)

Cholesky factorizations

$$B = P L L^T P^T$$

- Reordering only once to get P
 - One numerical factorization per interior-point iteration $O(n^3)$
 - Forward/backward substitution twice per iteration $O(n^2)$
- Per-iteration complexity**
 $O(n^3)$

Convergence

Mehrotra's algorithm

No convergence theory \longrightarrow Examples where it **diverges** (rare!)

Fantastic convergence **in practice** \longrightarrow Less than 30 iterations

Theoretical iteration complexity

Alternative versions (slower than Mehrotra) converge in $O(\sqrt{n})$ iterations



Floating point operations

$$O(n^{3.5})$$

Average iteration complexity

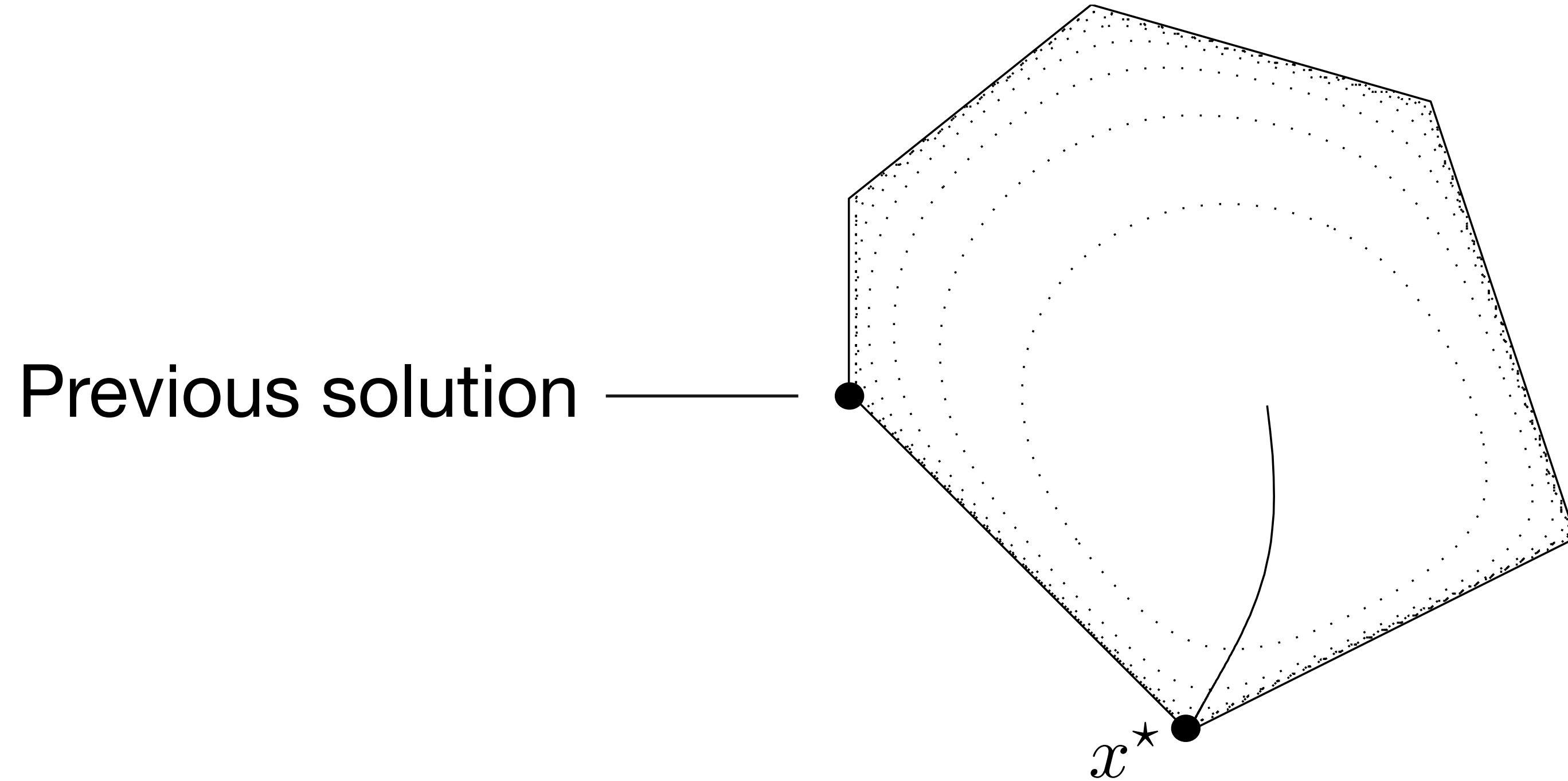
Average iterations complexity is $O(\log n)$



$$O(n^3 \log n)$$

Warm-starting

Interior-point methods are **difficult to warm-start**



Badly centered
initial point



Hard to make progress
with long steps

Homogeneous self-dual embedding

Optimality conditions

Primal

$$\begin{aligned} &\text{minimize} && c^T x \\ &\text{subject to} && Ax + s = b \\ &&& s \geq 0 \end{aligned}$$

Dual

$$\begin{aligned} &\text{maximize} && -b^T y \\ &\text{subject to} && A^T y + c = 0 \\ &&& y \geq 0 \end{aligned}$$

Optimality conditions

$$\begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & A^T \\ -A & 0 \\ c^T & b^T \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ b \\ 0 \end{bmatrix}$$

$$s, y \geq 0$$

Any (x^*, s^*, y^*) satisfying these conditions is **optimal**

What happens if the problem is infeasible?

How do you detect infeasibility/unboundedness?

Primal

$$\begin{aligned} &\text{minimize} && c^T x \\ &\text{subject to} && Ax + s = b \\ &&& s \geq 0 \end{aligned}$$

Dual

$$\begin{aligned} &\text{maximize} && -b^T y \\ &\text{subject to} && A^T y + c = 0 \\ &&& y \geq 0 \end{aligned}$$

Alternatives (Farkas lemma) Write feasibility problem and dualize...

- **primal feasible:** $Ax + s = b, \quad s \geq 0$
- **primal infeasible:** $A^T \underline{y} = 0, \quad b^T \underline{y} < 0, \quad \underline{y} \geq 0$ (primal infeasibility certificate)
- **dual feasible:** $A^T y + c = 0, \quad y \geq 0$
- **dual infeasible:** $A \underline{x} \leq 0, \quad c^T \underline{x} < 0$ (dual infeasibility certificate)

The homogeneous self-dual embedding

Derivation

Introduce two new variables $\kappa, \tau \geq 0$

Homogeneous self-dual embedding

$$\begin{bmatrix} 0 \\ s \\ \kappa \end{bmatrix} = \begin{bmatrix} 0 & A^T & c \\ -A & 0 & b \\ -c^T & -b^T & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \tau \end{bmatrix}$$

$$s, y, \kappa, \tau \geq 0$$



$$Qu = v$$

$$u, v \geq 0$$

$$Q = \begin{bmatrix} 0 & A^T & c \\ -A & 0 & b \\ -c^T & -b^T & 0 \end{bmatrix}$$

$$u = (x, y, \tau)$$

$$v = (0, s, \kappa)$$

The homogeneous self-dual embedding

Properties

$$Qu = v$$

$$u, v \geq 0$$

$$Q = \begin{bmatrix} 0 & A^T & c \\ -A & 0 & b \\ -c^T & -b^T & 0 \end{bmatrix}$$

$$u = (x, y, \tau)$$

$$v = (0, s, \kappa)$$

Matrix

• Q is skew-symmetric: $Q^T = -Q \Rightarrow u^T Qu = 0$

• $u \perp v$ **proof** $Qu - v = 0 \Rightarrow u^T Qu - u^T v = 0 \Rightarrow u^T v = 0$ ■

Homogeneous

(u, v) satisfy $Qu = v, (v, u) \geq 0 \Rightarrow \alpha(u, v)$ with $\alpha \geq 0$ feasible

Always feasible

$\alpha = 0 \Rightarrow (0, 0)$ is feasible

Self-dual problem

$$\begin{array}{ll} \text{minimize} & 0 \\ \text{subject to} & Qu = v \\ & u, v \geq 0 \end{array} \quad Q \text{ skew-symmetric: } Q^T = -Q$$

The dual is identical to the primal

Proof

$$g(\nu, \lambda, \mu) = \underset{u, v}{\text{minimize}} \mathcal{L}(u, v, \nu, \lambda, \mu) = \nu^T (Qu - v) - \lambda^T u - \mu^T v$$

$$\frac{\partial \mathcal{L}}{\partial u} = Q^T \nu - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial v} = -\nu^T - \mu = 0 \quad \Rightarrow \quad \nu = -\mu$$

Dual

$$\begin{array}{ll} \text{minimize} & 0 \\ \text{subject to} & Q\mu = \lambda \\ & \mu, \lambda \geq 0 \end{array}$$



From self-duality to strict complementarity

Primal

$$\begin{aligned} &\text{minimize} && 0 \\ &\text{subject to} && Qu = v \\ &&& u, v \geq 0 \end{aligned}$$

Dual

$$\begin{aligned} &\text{minimize} && 0 \\ &\text{subject to} && Q\mu = \lambda \\ &&& \mu, \lambda \geq 0 \end{aligned}$$

$$\begin{aligned} u &= (x, y, \tau) \\ v &= (0, s, \kappa) \end{aligned}$$

LP strict complementarity

$$\begin{aligned} u^T \lambda &= 0, & u + \lambda &> 0 \\ v^T \mu &= 0, & v + \mu &> 0 \end{aligned}$$

→

Self-dual

$$u = \mu, \quad v = \lambda$$

→

Strict complementarity

$$\begin{aligned} u + v &> 0 &\Rightarrow & \begin{aligned} y + s &> 0 \\ \tau + \kappa &> 0 \end{aligned} \end{aligned}$$

The homogeneous self-dual embedding

Outcomes

Find x, s, y, κ, τ such that

$$\begin{bmatrix} 0 \\ s \\ \kappa \end{bmatrix} = \begin{bmatrix} 0 & A^T & c \\ -A & 0 & b \\ -c^T & -b^T & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \tau \end{bmatrix}$$
$$s, y, \kappa, \tau \geq 0$$

Note. By strict complementarity, we can ensure $\kappa + \tau > 0$

Case 1: feasibility

$\tau > 0, \kappa = 0$ define $(\hat{x}, \hat{s}, \hat{y}) = (x^*/\tau, s^*/\tau, y^*/\tau)$

$$\begin{aligned} 0 &= A^T \hat{y} + c & \hat{s} &\geq 0, & \hat{y} &\geq 0, & \hat{s}^T \hat{y} &= 0 \\ \hat{s} &= -A\hat{x} + b \end{aligned}$$

→ $(\hat{x}, \hat{s}, \hat{y})$ is a **solution** to the original problem

The homogeneous self-dual embedding

Outcomes

$$\text{Find } x, s, y, \kappa, \tau \text{ such that } \begin{bmatrix} 0 \\ s \\ \kappa \end{bmatrix} = \begin{bmatrix} 0 & A^T & c \\ -A & 0 & b \\ -c^T & -b^T & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \tau \end{bmatrix}$$
$$s, y, \kappa, \tau \geq 0$$

Case 2: infeasibility

$\tau = 0, \kappa > 0 \longrightarrow c^T x + b^T y < 0$ (**impossible**). Must have infeasibility

If $b^T y < 0$ then $\hat{y} = y / (-b^T y)$ is a **certificate of primal infeasibility**

$$A^T \hat{y} = 0, \quad b^T \hat{y} = -1 < 0, \quad \hat{y} \geq 0$$

If $c^T x < 0$ then $\hat{x} = x / (-c^T x)$ is a **certificate of dual infeasibility**

$$A \hat{x} \leq 0, \quad c^T \hat{x} = -1 < 0$$

Interior-point method for homogeneous self-dual embedding

Linear complementarity problem

$$Qu = v$$

$$u^T v = 0$$

$$u, v \geq 0$$

Equations

$$h(u, v) = \begin{bmatrix} Qu - v \\ UV\mathbf{1} \end{bmatrix} = 0$$

$$u, v \geq 0$$

Directions

$$\begin{bmatrix} Q & -I \\ V & U \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix} = \begin{bmatrix} -r_e \\ -UV\mathbf{1} + \sigma\mu\mathbf{1} \end{bmatrix}$$

$$r_e = Qu - v$$

$$\mu = (u^T v) / d$$

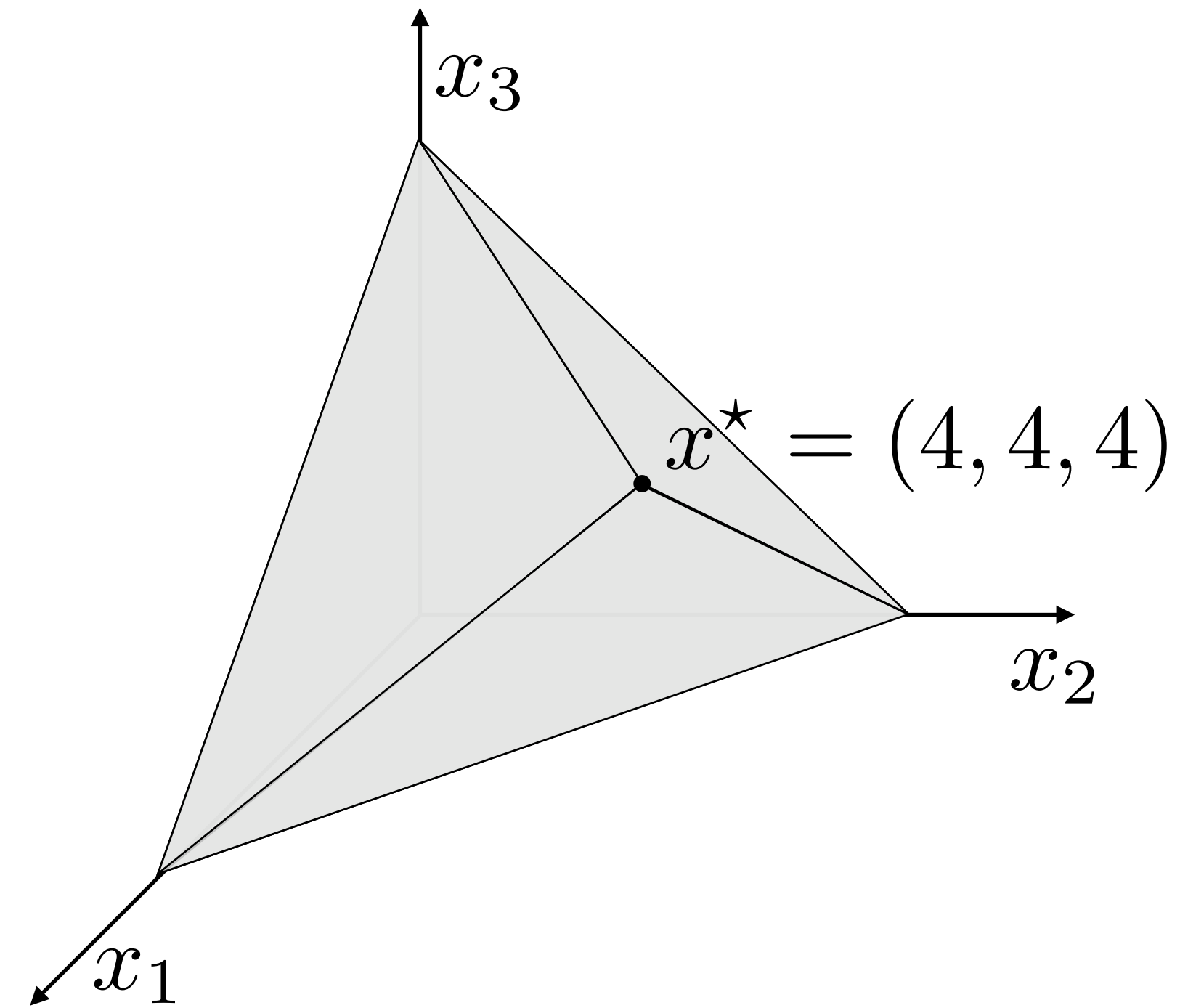
Line search to enforce $u, v > 0$

$$(u, v) \leftarrow (u, v) + \alpha(\Delta u, \Delta v)$$

Interior-point vs simplex

Example

$$\begin{aligned} &\text{minimize} && -10x_1 - 12x_2 - 12x_3 \\ &\text{subject to} && x_1 + 2x_2 + 2x_3 \leq 20 \\ &&& 2x_1 + x_2 + x_3 \leq 20 \\ &&& 2x_1 + 2x_2 + x_3 \leq 20 \\ &&& x_1, x_2, x_3 \geq 0 \end{aligned}$$



$$\begin{aligned} &\text{minimize} && c^T x \\ &\text{subject to} && Ax \leq b \\ &&& x \geq 0 \end{aligned}$$

$$c = (-10, -12, -12)$$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$b = (20, 20, 20)$$

Example with real solver

CVXOPT (open-source)

Code

```
import numpy as np
import cvxpy as cp

c = np.array([-10, -12, -12])
A = np.array([[1, 2, 2],
              [2, 1, 2],
              [2, 2, 1]])
b = np.array([20, 20, 20])
n = len(c)

x = cp.Variable(n)
problem = cp.Problem(cp.Minimize(c @ x),
                    [A @ x <= b, x >= 0])
problem.solve(solver=cp.CVXOPT, verbose=True)
```

Output

```
      pcost      dcost      gap      pres      dres      k/t
0: -1.3077e+02 -2.3692e+02 2e+01 1e-16 6e-01 1e+00
1: -1.3522e+02 -1.4089e+02 1e+00 2e-16 3e-02 4e-02
2: -1.3599e+02 -1.3605e+02 1e-02 2e-16 3e-04 4e-04
3: -1.3600e+02 -1.3600e+02 1e-04 1e-16 3e-06 4e-06
4: -1.3600e+02 -1.3600e+02 1e-06 1e-16 3e-08 4e-08
Optimal solution found.
```

Solution

```
In [3]: x.value
Out[3]: array([3.99999999, 4.          , 4.          ])
```

Average interior-point complexity

Random LPs

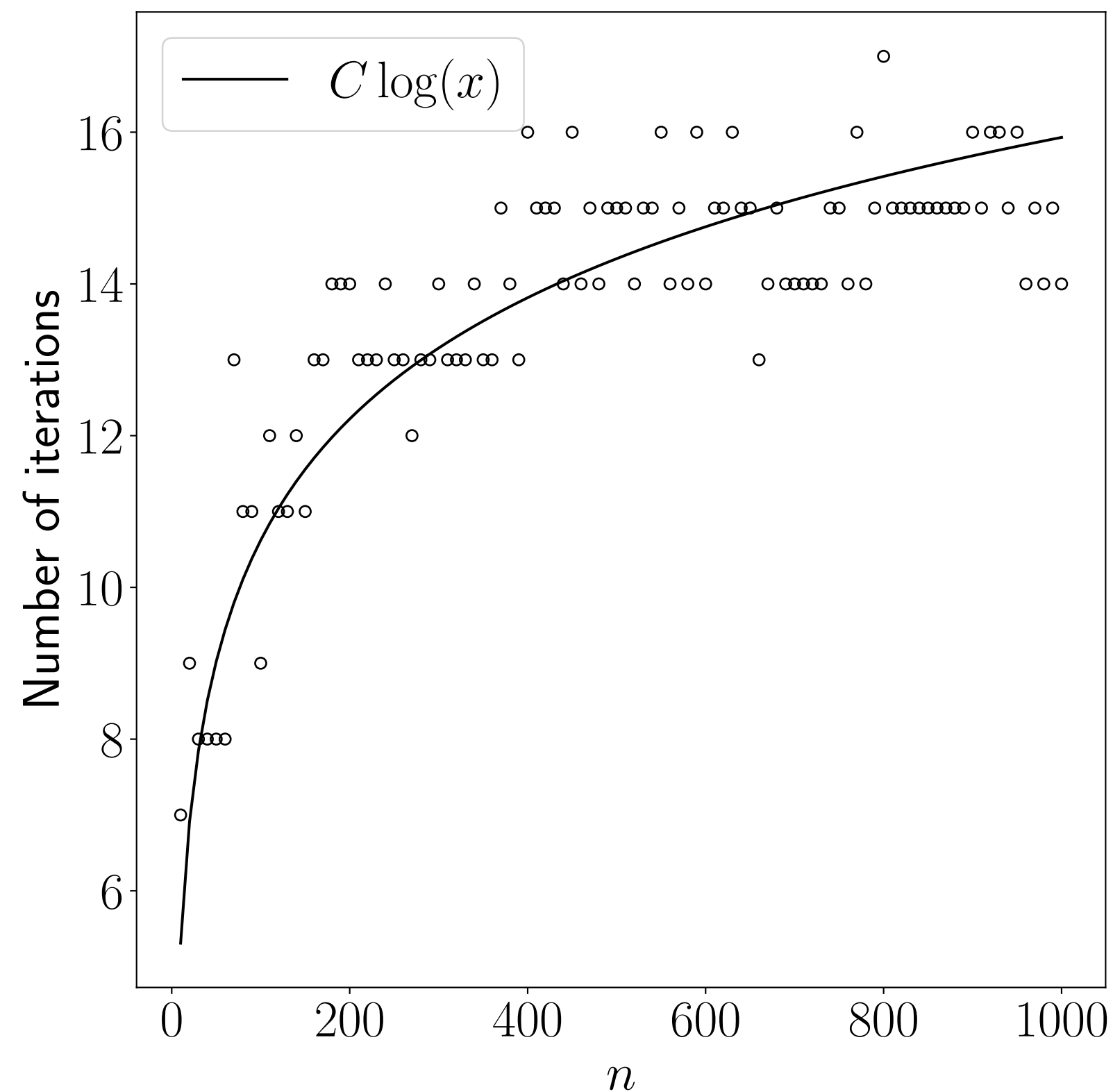
minimize $c^T x$

n variables

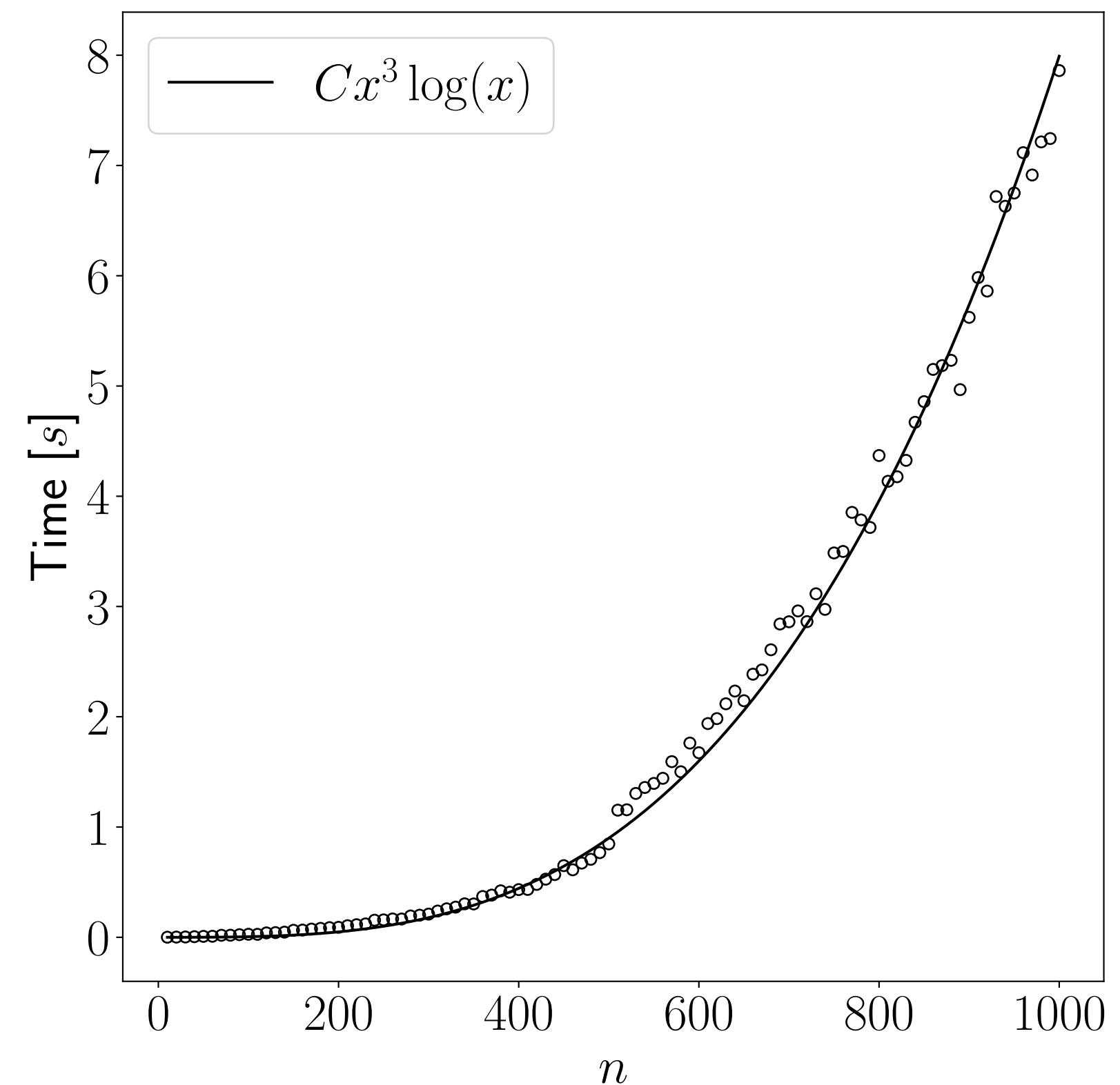
subject to $Ax \leq b$

$3n$ constraints

Iterations: $O(\log n)$



Time: $O(n^3 \log n)$



Comparison between interior-point method and simplex

Primal simplex

- Primal feasibility
- Zero duality gap



Dual feasibility

Dual simplex

- Dual feasibility
- Zero duality gap



Primal feasibility

Primal-dual interior-point

- Interior condition



- Primal feasibility
- Dual feasibility
- Zero duality gap

Exponential worst-case complexity

Requires feasible point

Can be warm-started

Polynomial worst-case complexity

Allows infeasible start

Cannot be warm-started

Which algorithm should I use?

Dual simplex

- Small-to-medium problems
- Repeated solves with varying data

Interior-point (barrier)

- Medium-to-large problems
- Sparse structured problems

How do solvers with multiple options decide?

Concurrent Optimization

Why not both? (crossover)

Interior-point → Few simplex steps

Interior-point methods implementation

Today, we learned to:

- **Apply** Mehrotra predictor-corrector algorithm
- **Exploit** linear algebra to speedup computations
- **Detect** infeasibility/unboundedness with homogeneous self-dual embedding
- **Analyze** empirical complexity
- **Compare** interior-point and simplex methods

Next lecture

- Introduction to nonlinear optimization