High-Speed Integer Optimal Control using ADP

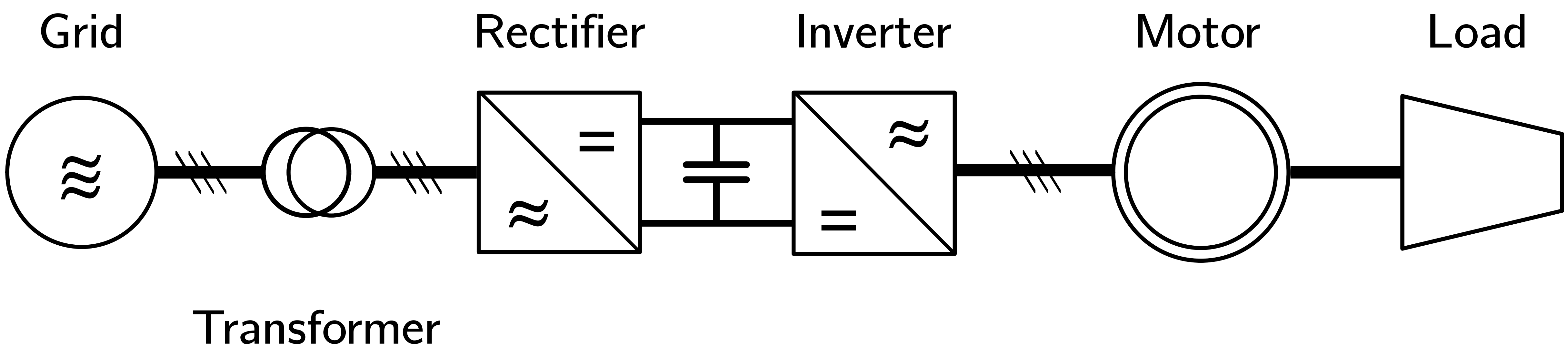
Bartolomeo Stellato and Paul Goulart

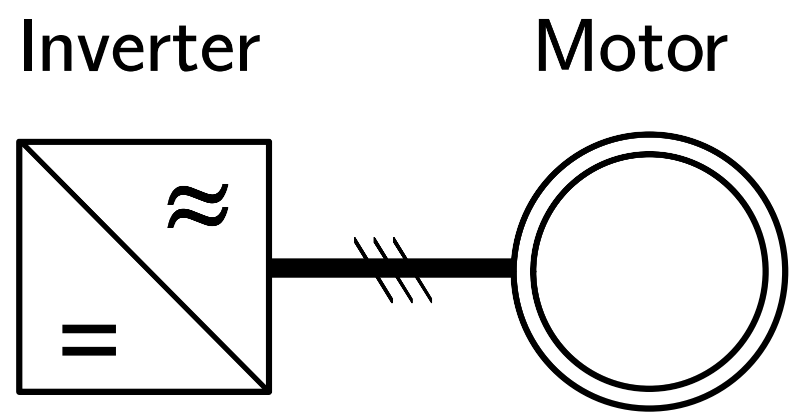
12 Jan, 2017 IMT Lucca, Italy

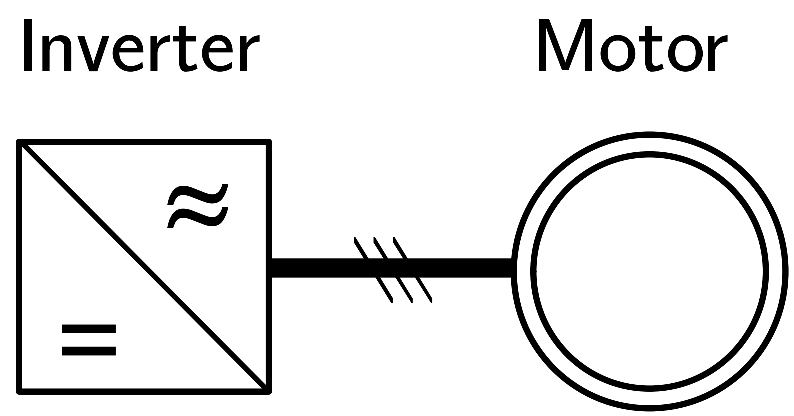
UNIVERSITY OF OXFORD


Medium-voltage drives market is worth 4bln $....
... and is growing 10% every year!

Power Distribution




Inverter
Motor
≈
=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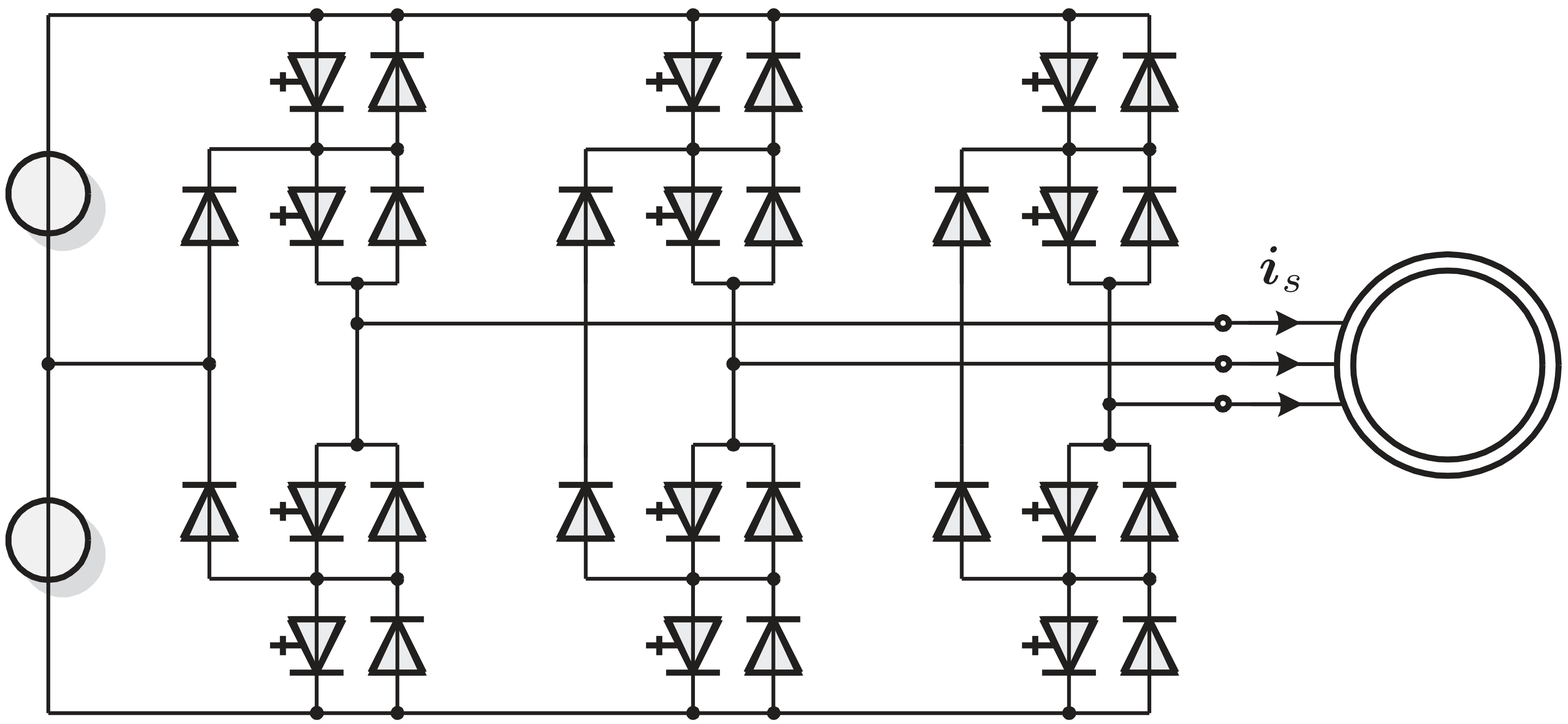
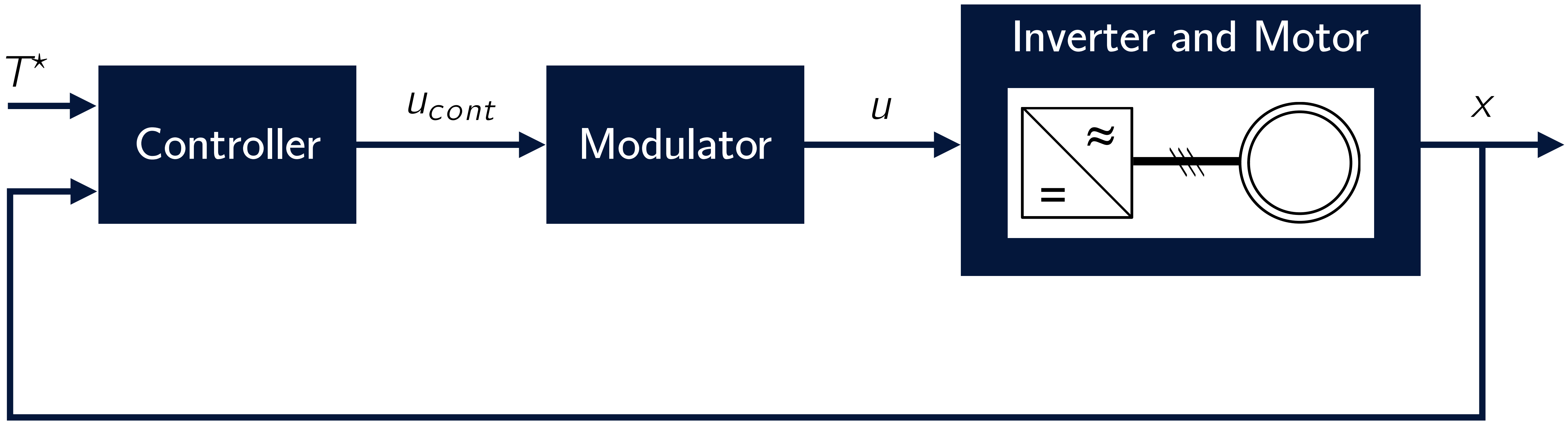
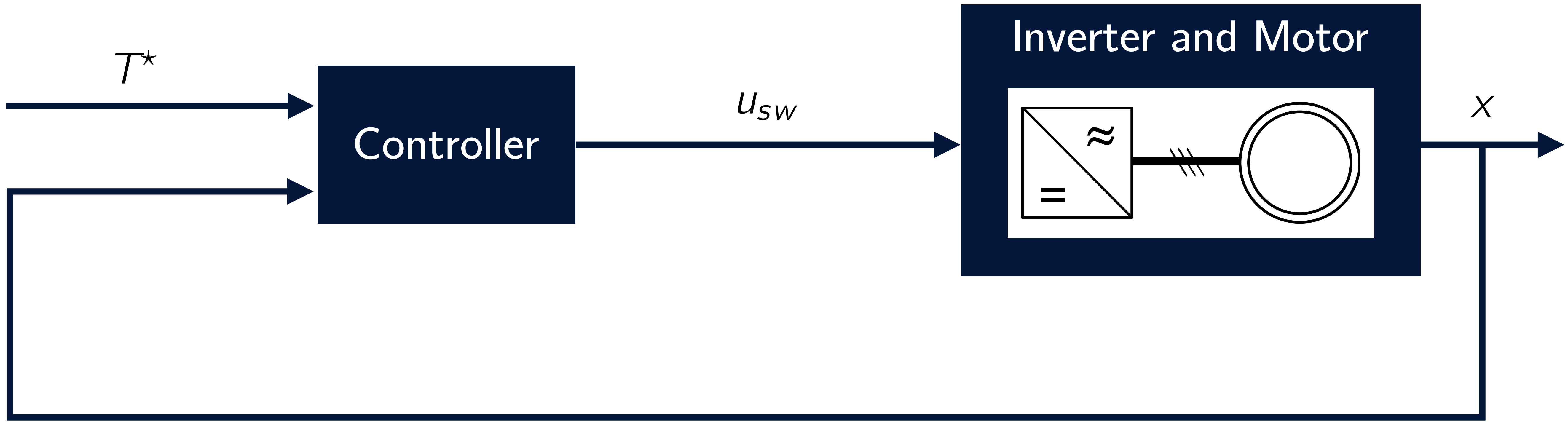

i_s

Traditional Control

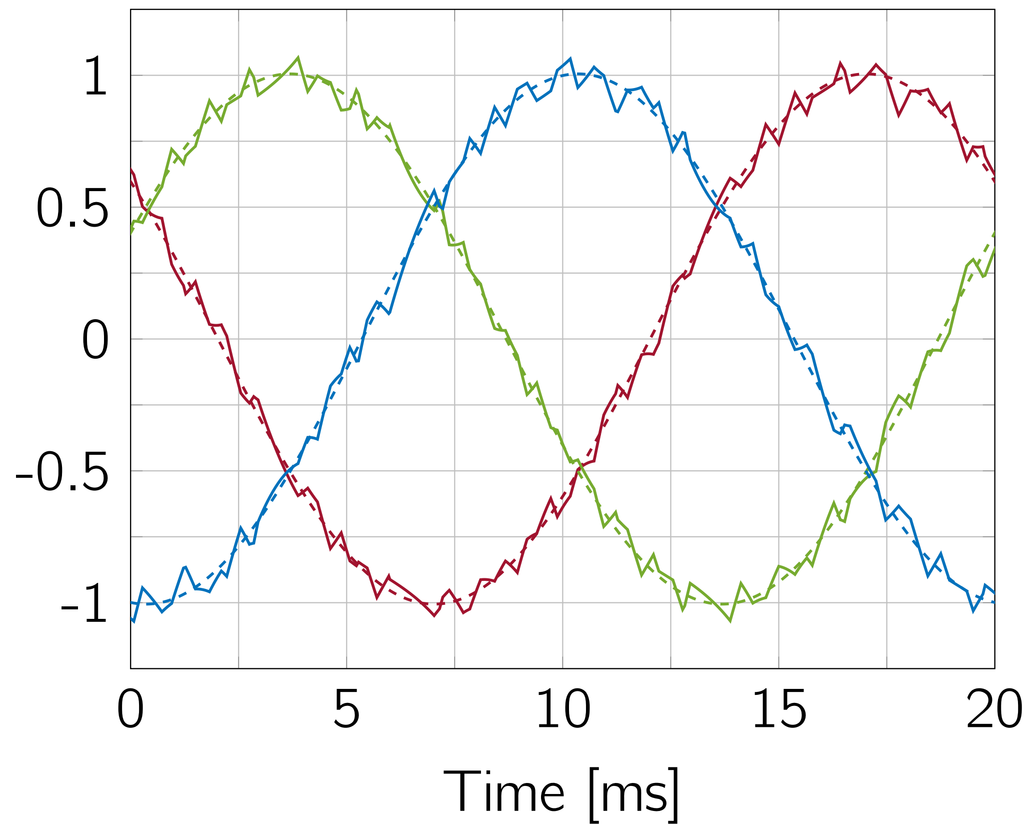


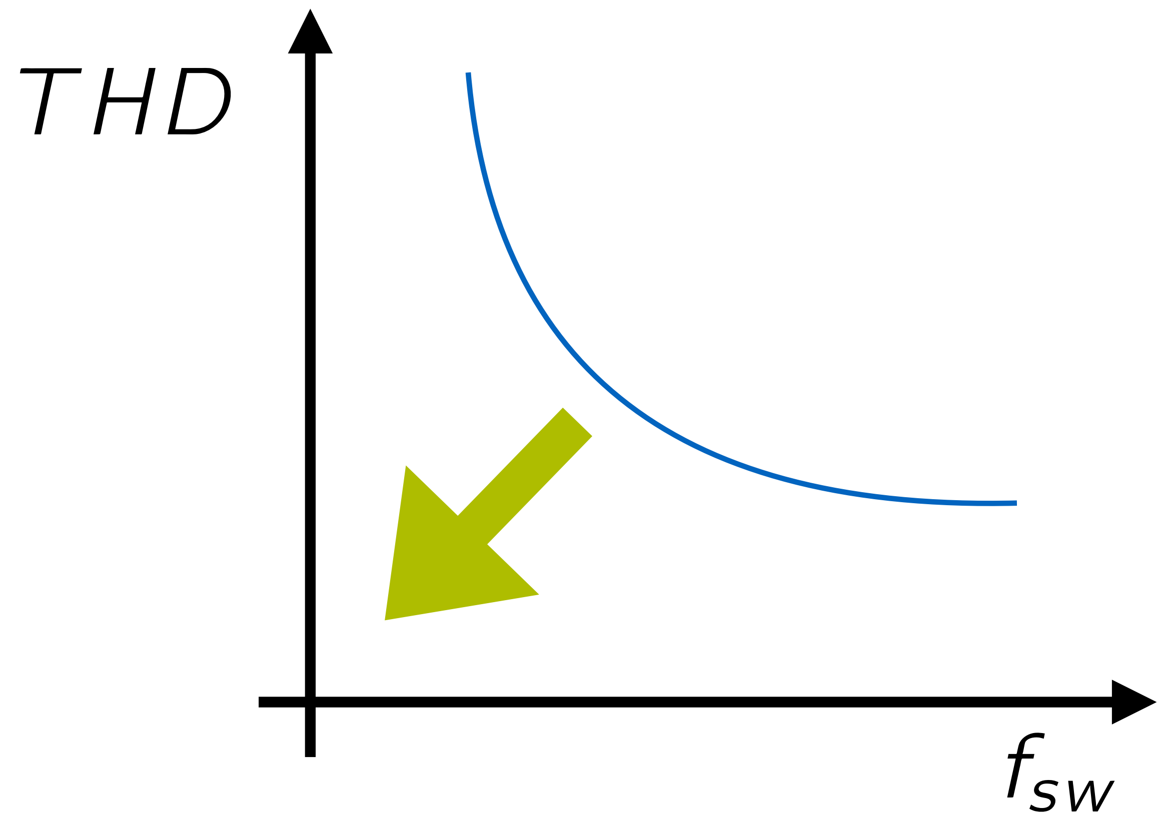
Direct MPC



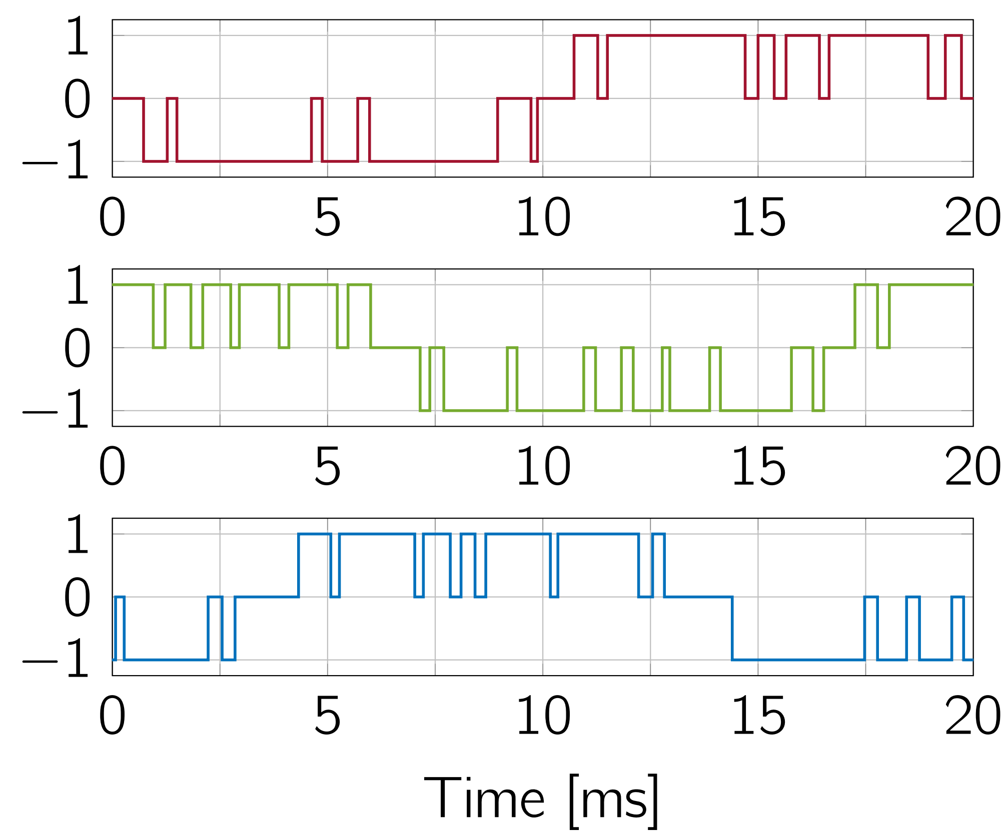
... But Direct MPC is Difficult

Currents

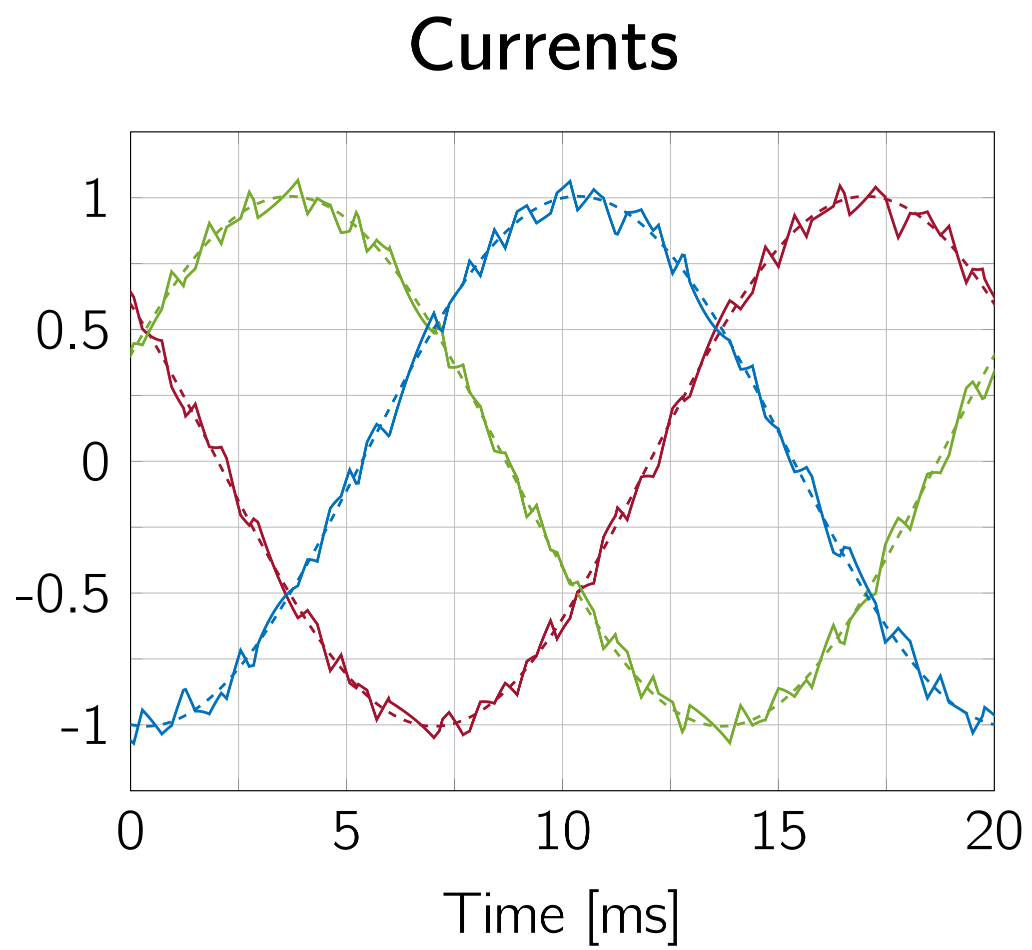


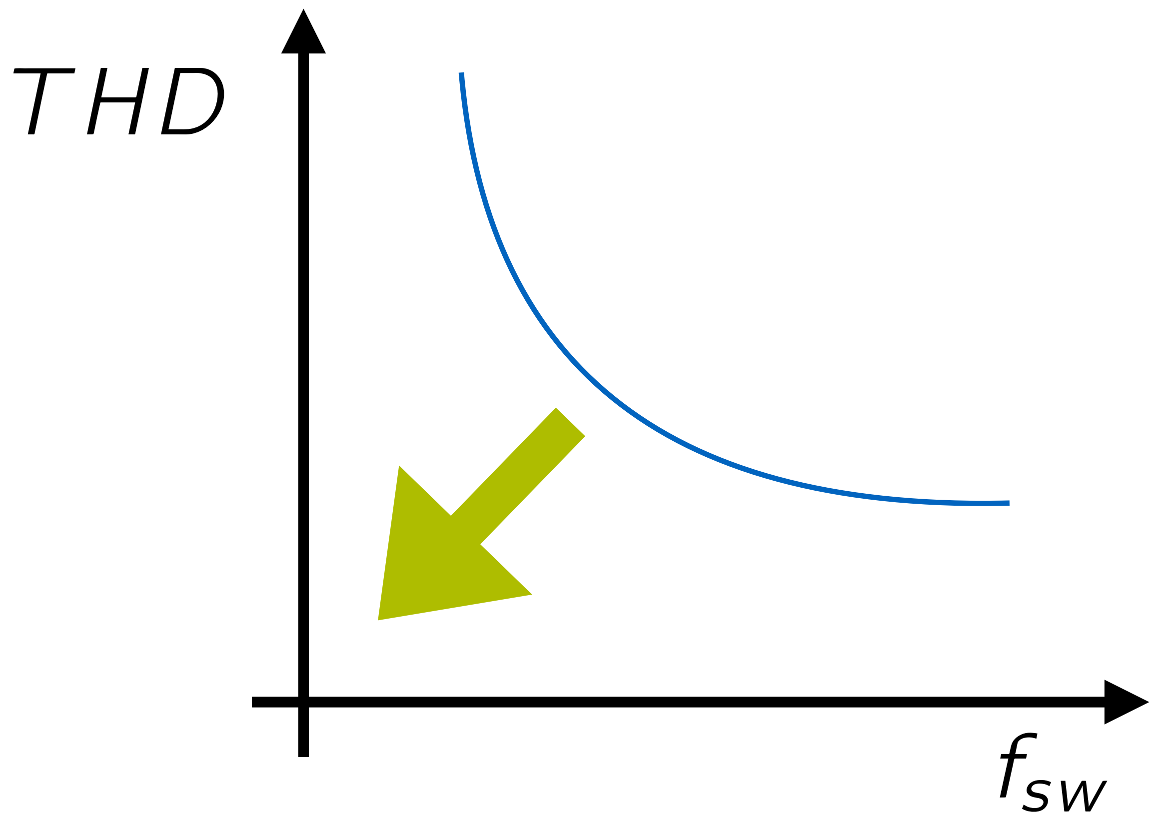


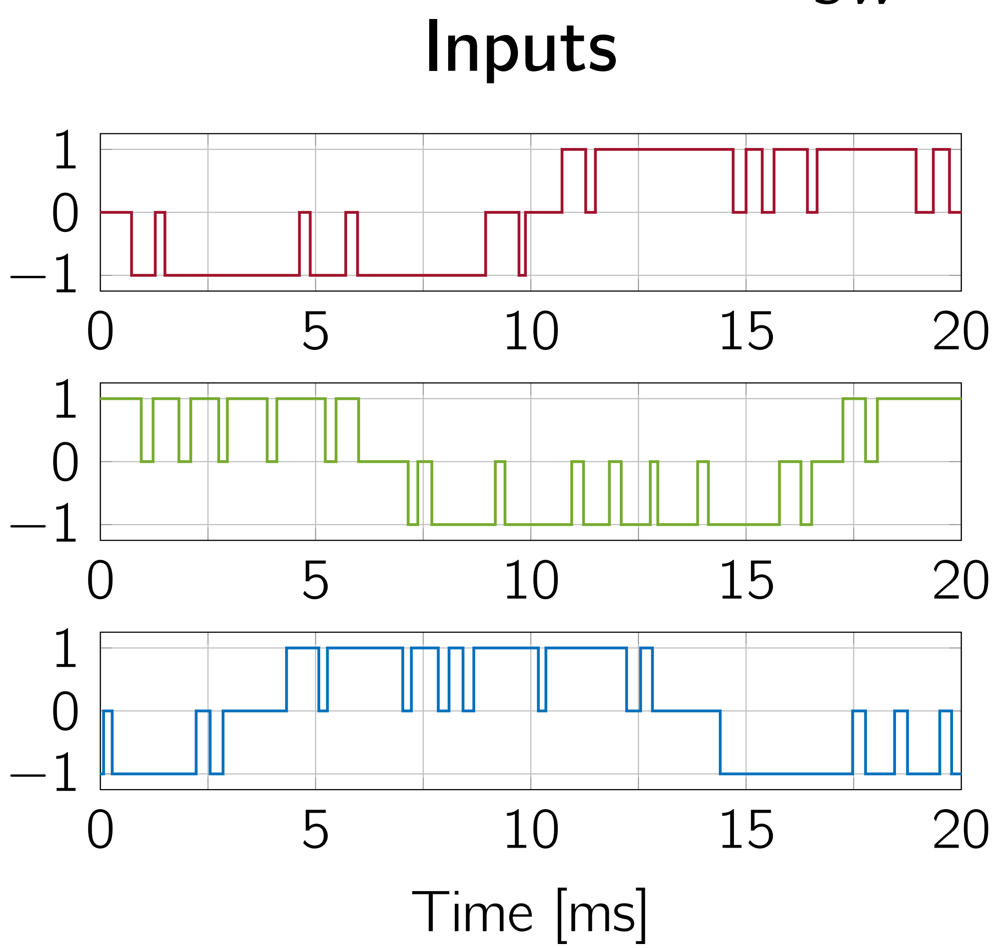
Inputs



... But Direct MPC is Difficult







Tradeoff
THD **vs** f_{sw}

Very fast dynamics

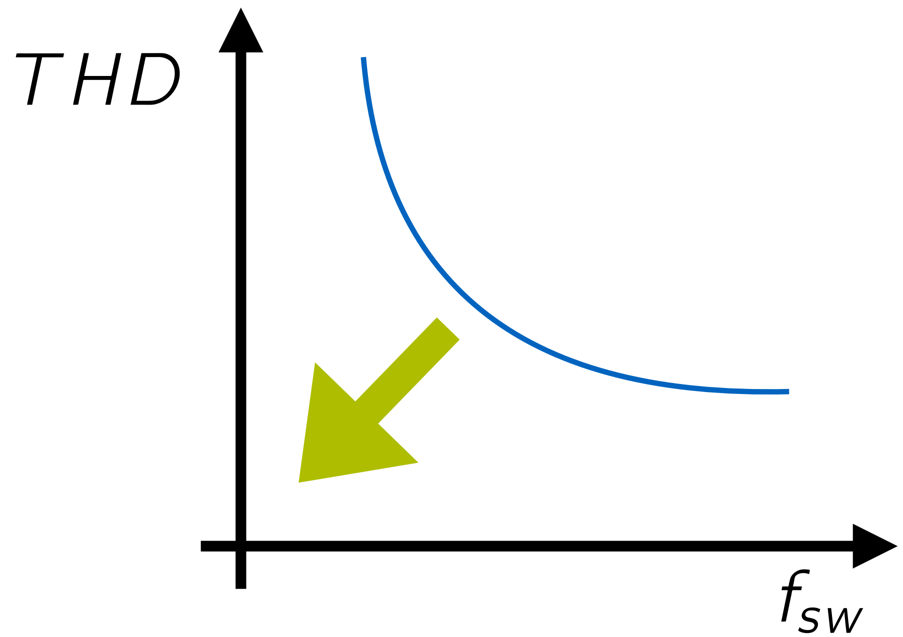
... But Direct MPC is Difficult

Very fast dynamics

↓

Mixed Integer Optimization Problems in 25 μs!

Control Objectives



Tradeoff

THD **vs** f_{sw}

Timing

Mixed Integer Optimization Problems in 25 μs!

Problem Formulation

Total Harmonic Distortion

$$THD \sim \sqrt{\frac{1}{M}\sum_{k=0}^{\infty}\|i(k) - i^{\star}(k)\|_2^2}$$

Total Harmonic Distortion

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THD Component in Cost Function

$$\sum_{k=0}^{\infty}\gamma^{k}\|i(k) - i^{\star}(k)\|_2^2$$

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Internal Motor States

x_m

Total Harmonic Distortion

$$THD \sim \sqrt{\frac{1}{M}\sum_{k=0}^{\infty} \|i(k) - i^{\star}(k)\|_2^2}$$

***THD* Component in Cost Function**

$$\sum_{k=0}^{\infty} \gamma^k \|i(k) - i^{\star}(k)\|_2^2$$

Internal Motor States x_m

Oscillating References x_{osc}

Switching Frequency

$$f_{sw} = \lim_{T \to \infty} \frac{1}{T} \sum_{k=0}^{T} \|u(k) - u(k-1)\|_1$$

$$f_{sw} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

Switching Frequency

$$f_{sw} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

$\longrightarrow$ Approximate with IIR Filter

Switching Frequency

$$f_{sw} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

Approximate with IIR Filter

f_{sw} Component in Cost Function

$$\delta \sum_{k=0}^{\infty} \gamma^k \|\hat{f}_{sw}(k) - f_{sw}^{\star}\|_2^2$$

$$f_{sw} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

→ Approximate with IIR Filter

f_{sw} Component in Cost Function

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Frequency Estimate

Switching Frequency

$$f_{sw} = \lim_{T \to \infty} \underbrace{\frac{1}{T} \sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

Approximate with IIR Filter

f_{sw} Component in Cost Function

$$\delta \sum_{k=0}^{\infty} \gamma^k \|\hat{f}_{sw}(k) - f_{sw}^{\star}\|_2^2$$

Frequency Estimate

Desired Frequency

Switching Frequency

$$f_{sw} = \lim_{T\to\infty} \underbrace{\frac{1}{T}\sum_{k=0}^{T} \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

Approximate with IIR Filter

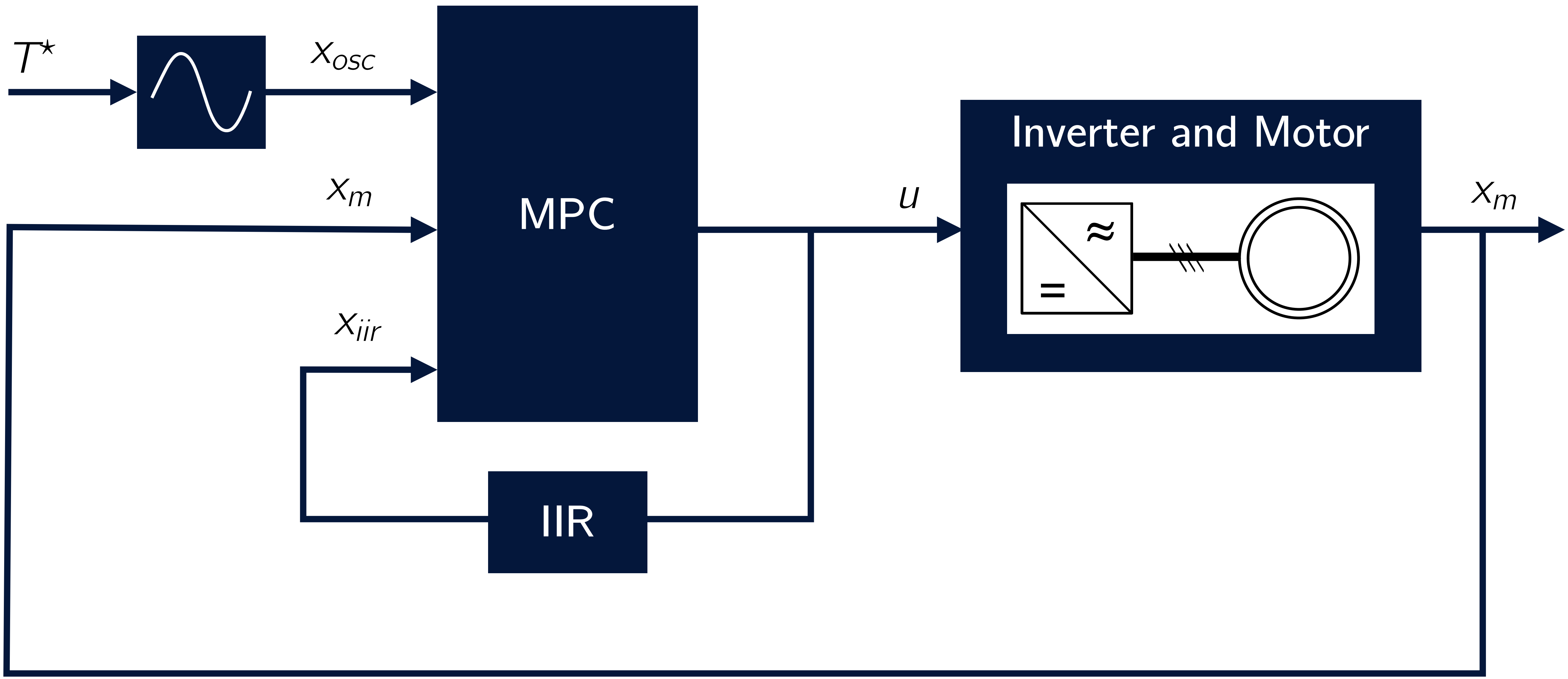
f_{sw} Component in Cost Function

$$\delta \sum_{k=0}^{\infty} \gamma^k \|\hat{f}_{sw}(k) - f_{sw}^\star\|_2^2$$

Frequency Estimate

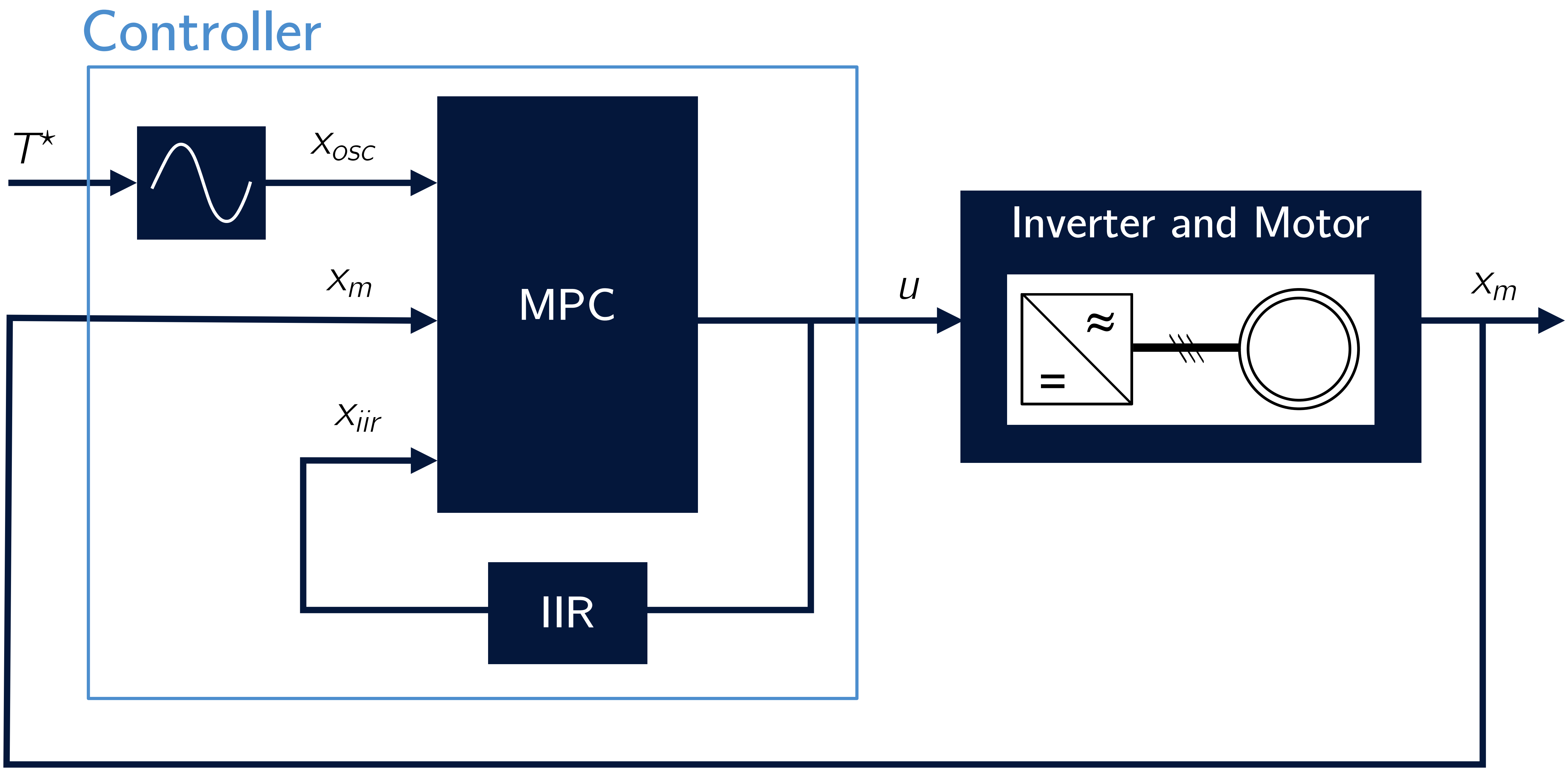
Desired Frequency

Filter States x_{iir}



States: $x = \begin{bmatrix} x_{osc}^\top & x_{im}^\top & x_{iir}^\top \end{bmatrix}^\top$

Complete Block Diagram



States: $x = \begin{bmatrix} x_{osc}^\top & x_{im}^\top & x_{iir}^\top \end{bmatrix}^\top$

Infinite Horizon

$$
\begin{aligned}
\text{minimize} \quad & \sum_{k=0}^{\infty} \gamma^k l(x(k)) \\
\text{subject to} \quad & x(k+1) = Ax(k) + Bu(k) \\
& x(0) = x_0 \\
& x(k) \in \mathcal{X},\ u(k) \in \{-1, 0, 1\}^3
\end{aligned}
$$

Optimal Control Problem

Infinite Horizon

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\begin{array}{ll}
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\end{array}
$$

$$
\text{where} \quad l(x(k)) = \underbrace{\|i(k) - i^\star(k)\|_2^2}_{THD} + \delta\underbrace{\|\hat{f}_{sw}(k) - f_{sw}^\star\|_2^2}_{f_{sw}}
$$

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Impossible to Solve Online

Problem Solution

Optimal Control Problem

Infinite Horizon

$$
\begin{array}{ll}
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& x(k) \in \mathcal{X},\, u(k) \in \{-1, 0, 1\}^3
\end{array}
$$

Optimal Control Problem

Short Finite Horizon

$$
\begin{array}{ll}
\text{minimize} & \displaystyle\sum_{k=0}^{\boldsymbol{N-1}} \gamma^k l(x(k)) + \boldsymbol{\gamma^N V(x(k))} \\
\text{subject to} & x(k+1) = Ax(k) + Bu(k) \\
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Optimal Control Problem

Short Finite Horizon

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Very Short Horizon N

Optimal Control Problem

Short Finite Horizon

$$
\begin{aligned}
\text{minimize} \quad & \sum_{k=0}^{N-1} \gamma^k l(x(k)) + \boldsymbol{\gamma}^{N} \boldsymbol{V}(\boldsymbol{x}(\boldsymbol{k})) \\
\text{subject to} \quad & x(k+1) = Ax(k) + Bu(k) \\
& x(0) = x_0 \\
& x(k) \in \mathcal{X},\, u(k) \in \{-1, 0, 1\}^3
\end{aligned}
$$

Very Short Horizon N

Approximate $V(x(k))$ Offline

Bellman Equality

$$V^\star(x) = \min_u \left\{ l(x) + \gamma V^\star\left(Ax + Bu\right) \right\}$$

Bellman Equality

$$V^\star(x) = \underbrace{\min_u \left\{ l(x) + \gamma V^\star\left(Ax + Bu\right)\right\}}_{\mathcal{T}V^*}$$

Bellman Operator $\mathcal{T}$

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Bellman Operator $\mathcal{T}$

Contractive $\lim_{M\to\infty} \mathcal{T}^M V = V^\star$

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Bellman Operator $\mathcal{T}$

Contractive $\lim_{M \to \infty} \mathcal{T}^M V = V^\star$

Monotone $V_1 \leq V_2 \implies \mathcal{T}V_1 \leq \mathcal{T}V_2$

Bellman Inequality

$$V(x) \leq \underbrace{\min_u \{l(x) + \gamma V(Ax + Bu)\}}_{\mathcal{T}V}$$

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Bellman Operator $\mathcal{T}$

Contractive $\lim_{M \to \infty} \mathcal{T}^M V = V^\star$

Monotone $V_1 \leq V_2 \implies \mathcal{T} V_1 \leq \mathcal{T} V_2$

Sufficient condition for underestimating $V^\star$

$$V \leq \mathcal{T} V \implies V \leq V^\star$$

Infinite Dimensional LP

$$\begin{array}{ll} \text{maximize} & \displaystyle\int_{\mathcal{X}} V(x)c(\mathrm{d}x) \\ \text{subject to} & V(x) \leq \mathcal{T} V(x) \quad \forall x \end{array}$$

Infinite Dimensional LP

$$\begin{array}{ll} \text{maximize} & \displaystyle\int_{\mathcal{X}} V(x)c(\mathrm{d}x) \\ \text{subject to} & V(x) \leq \mathcal{T}^{M} V(x) \quad \forall x \end{array}$$

Iterated Bellman Inequality

Finding the Best Underestimator

Infinite Dimensional LP

$$
\begin{array}{ll}
\text{maximize} & \displaystyle\int_{\mathcal{X}} V(x)c(\mathrm{d}x) \\
\text{subject to} & V(x) \leq \mathcal{T}^{M} V(x) \quad \forall x
\end{array}
$$

Iterated Bellman Inequality

Restrict to Quadratic Functions

$$V(x) = x^\top P x + 2q^\top x + r$$

Infinite Dimensional LP

$$\begin{array}{ll} \text{maximize} & \int_{\mathcal{X}} V(x)c(\mathrm{d}x) \\ \text{subject to} & V(x) \le \mathcal{T}^{M} V(x) \quad \forall x \end{array}$$

Iterated Bellman Inequality

Restrict to Quadratic Functions

$$V(x) = x^\top P x + 2q^\top x + r$$

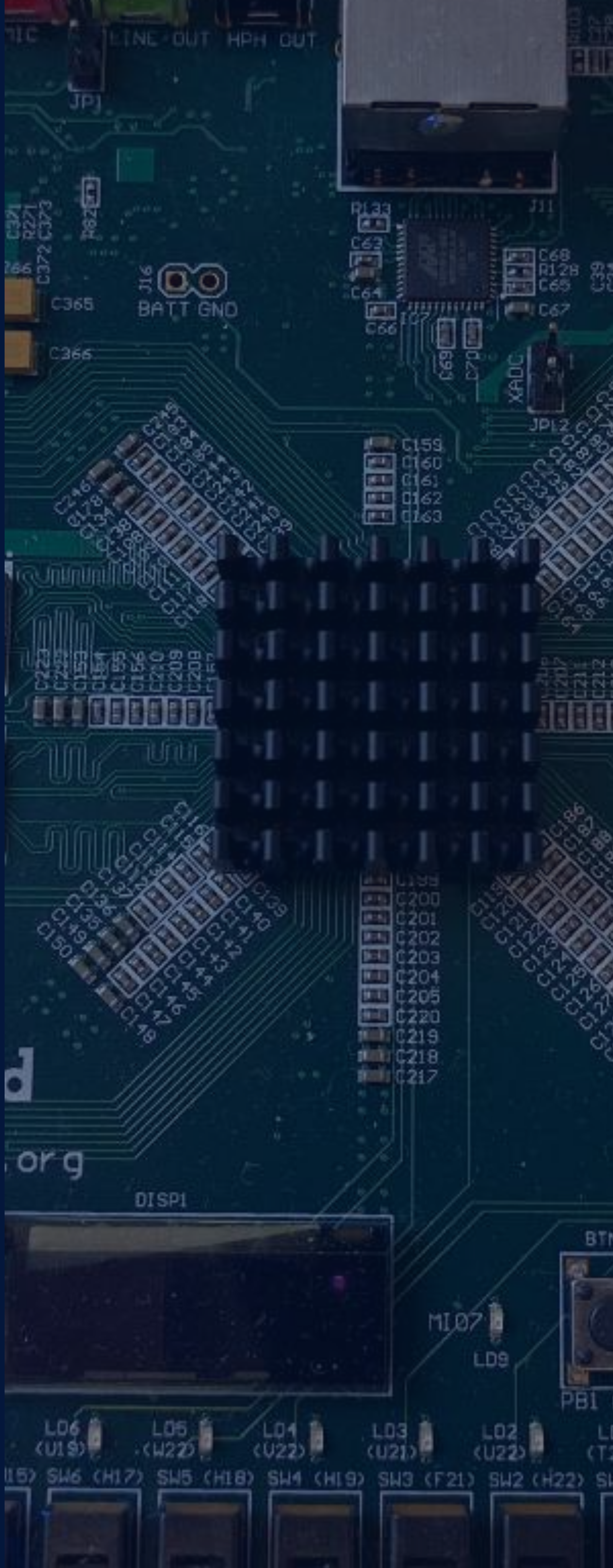
Tractable SDP (Offline)

FPGA Implementation

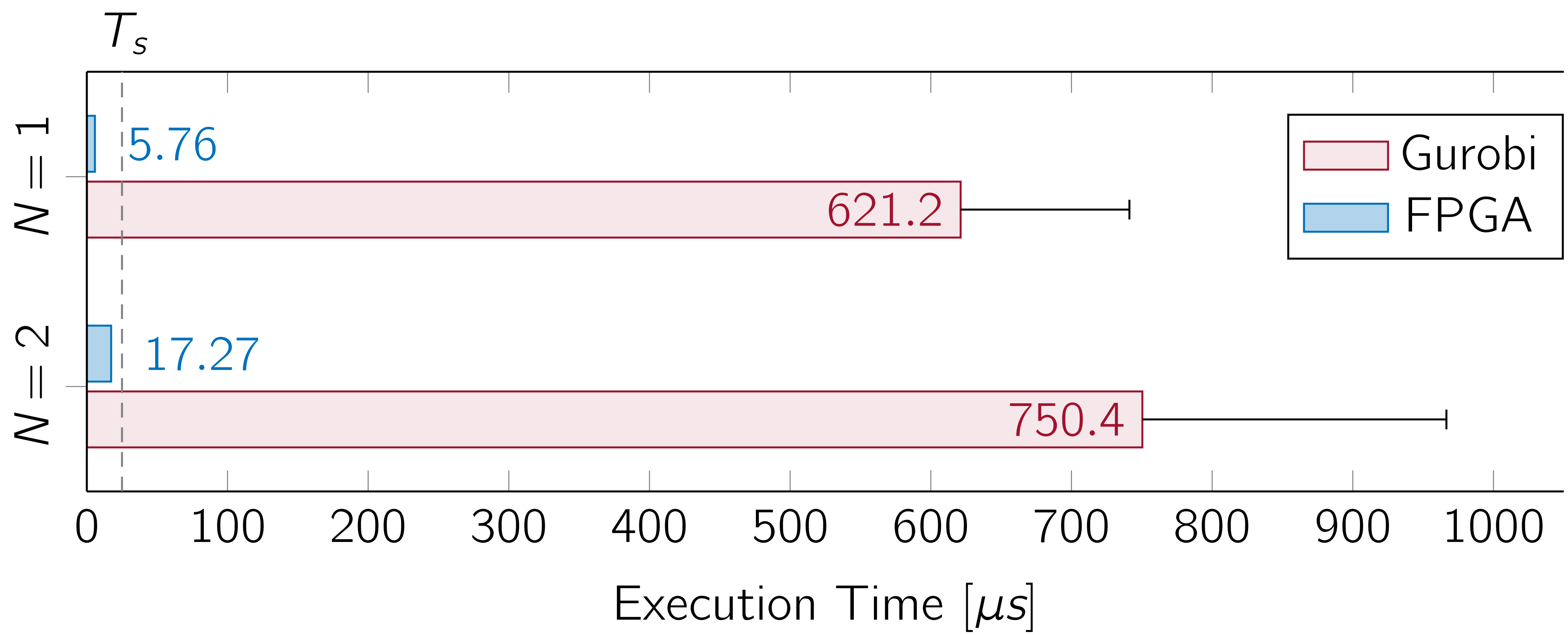
PROTOIP Toolbox

(Suardi et al.)

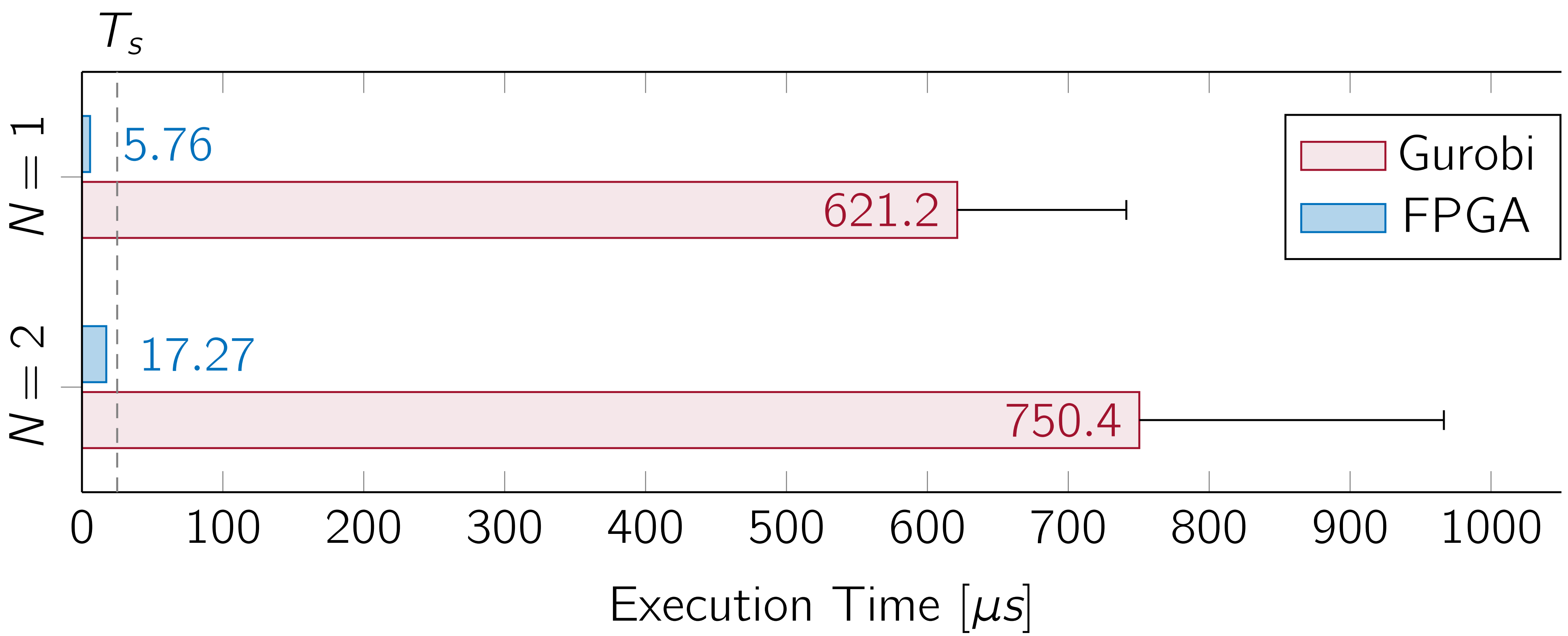
- Brute force enumeration
- Pipelined Evaluation of Switch Sequences
- Parallelized Matrix Computations



Timing Benchmarks



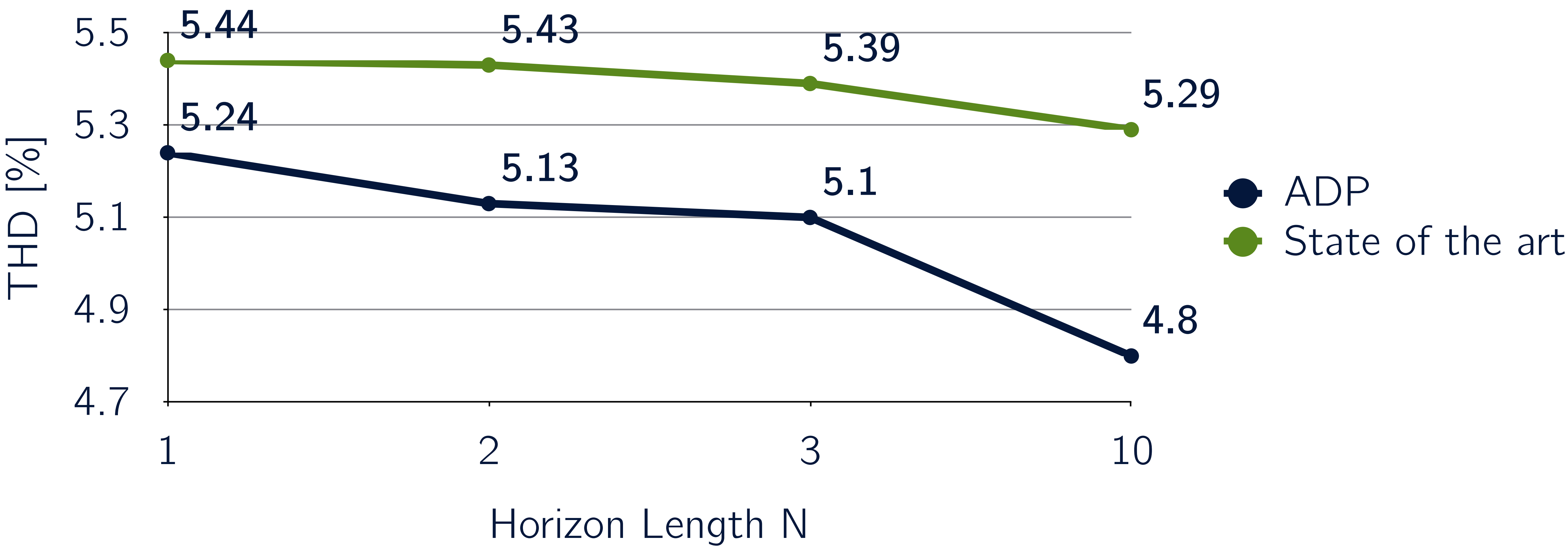
Timing Benchmarks



Computation Times under 25 μs!

Hardware in the Loop Tests

Steady-State Performance at $f_{sw} = 300$ Hz



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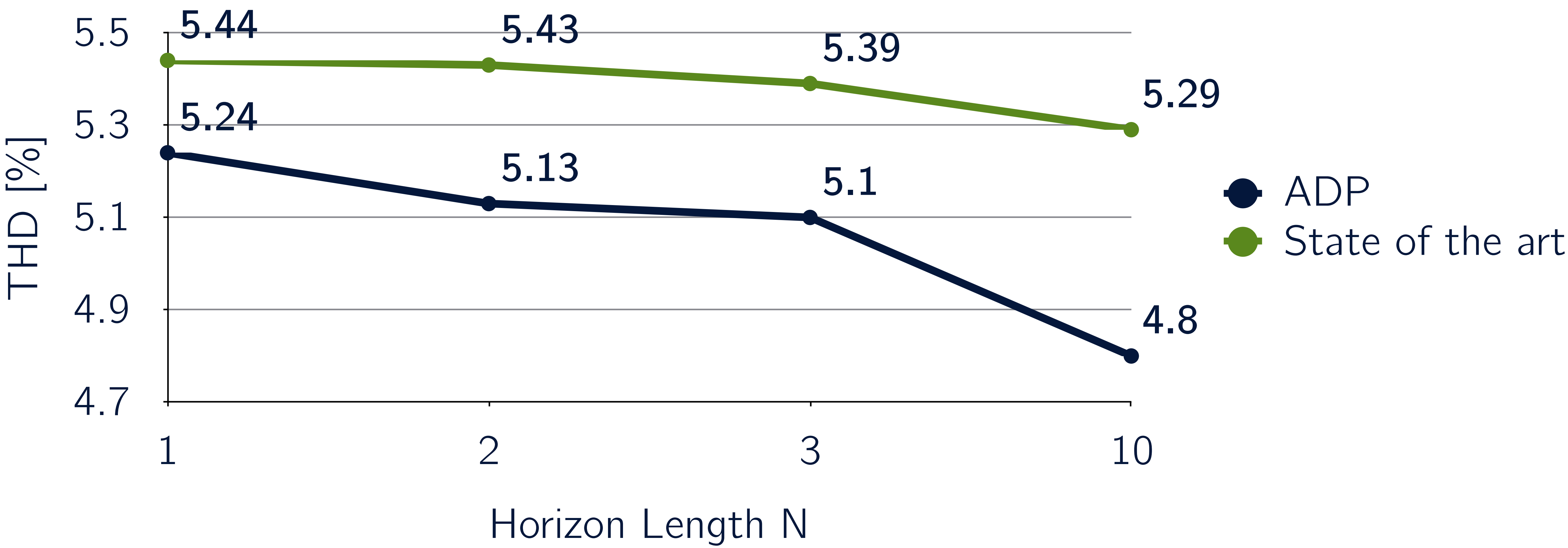
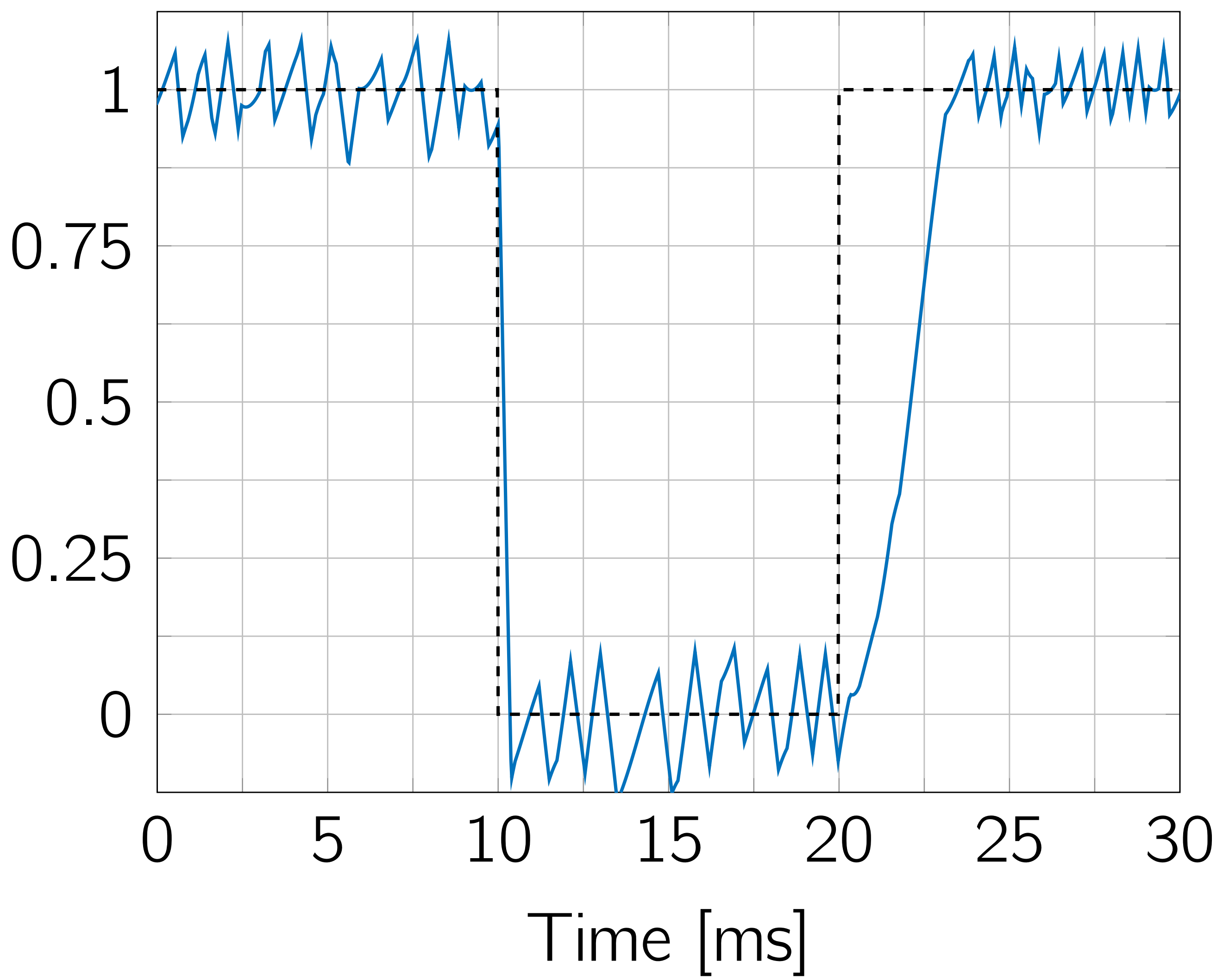


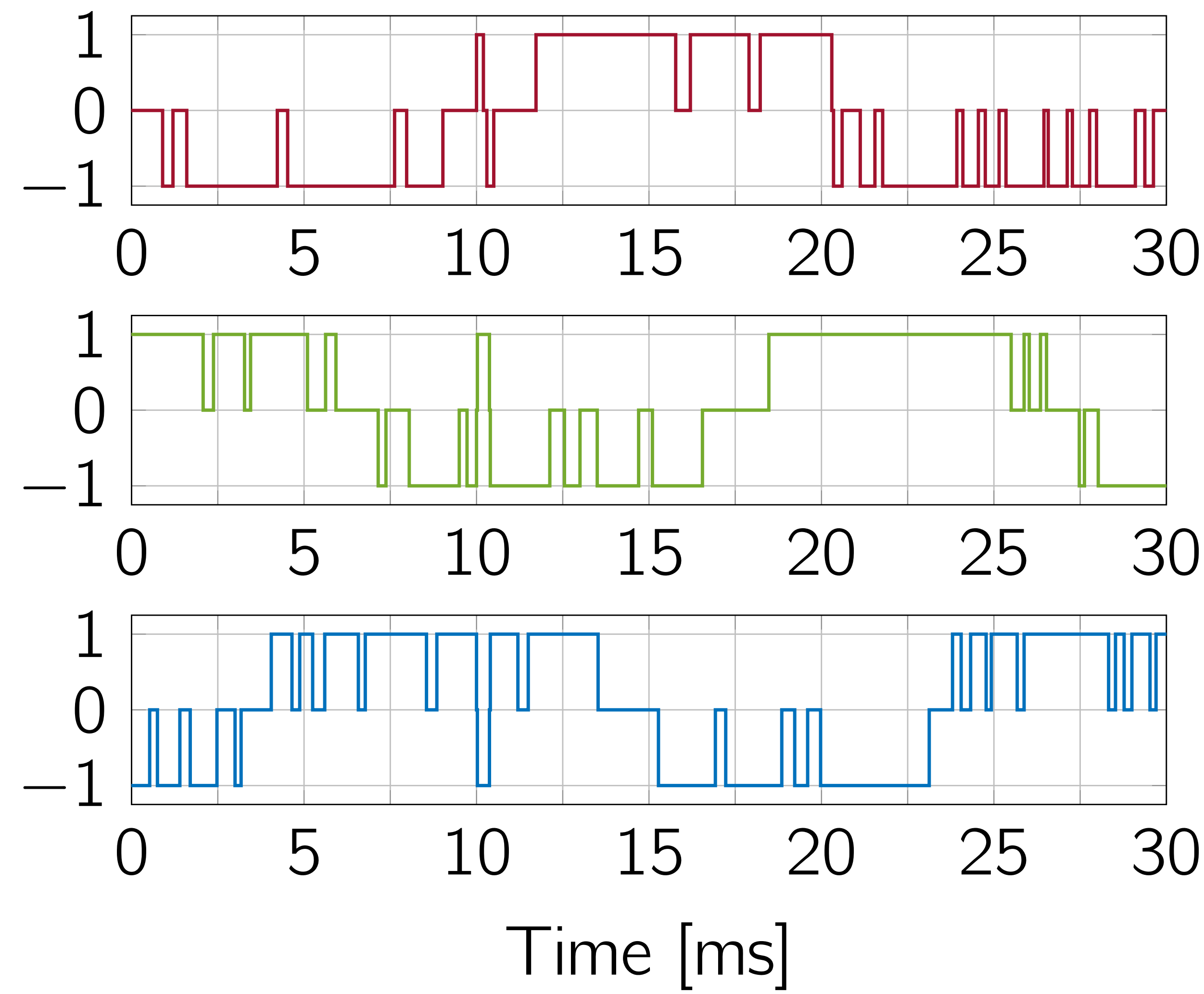
Better Performance with Short Horizons

Transients

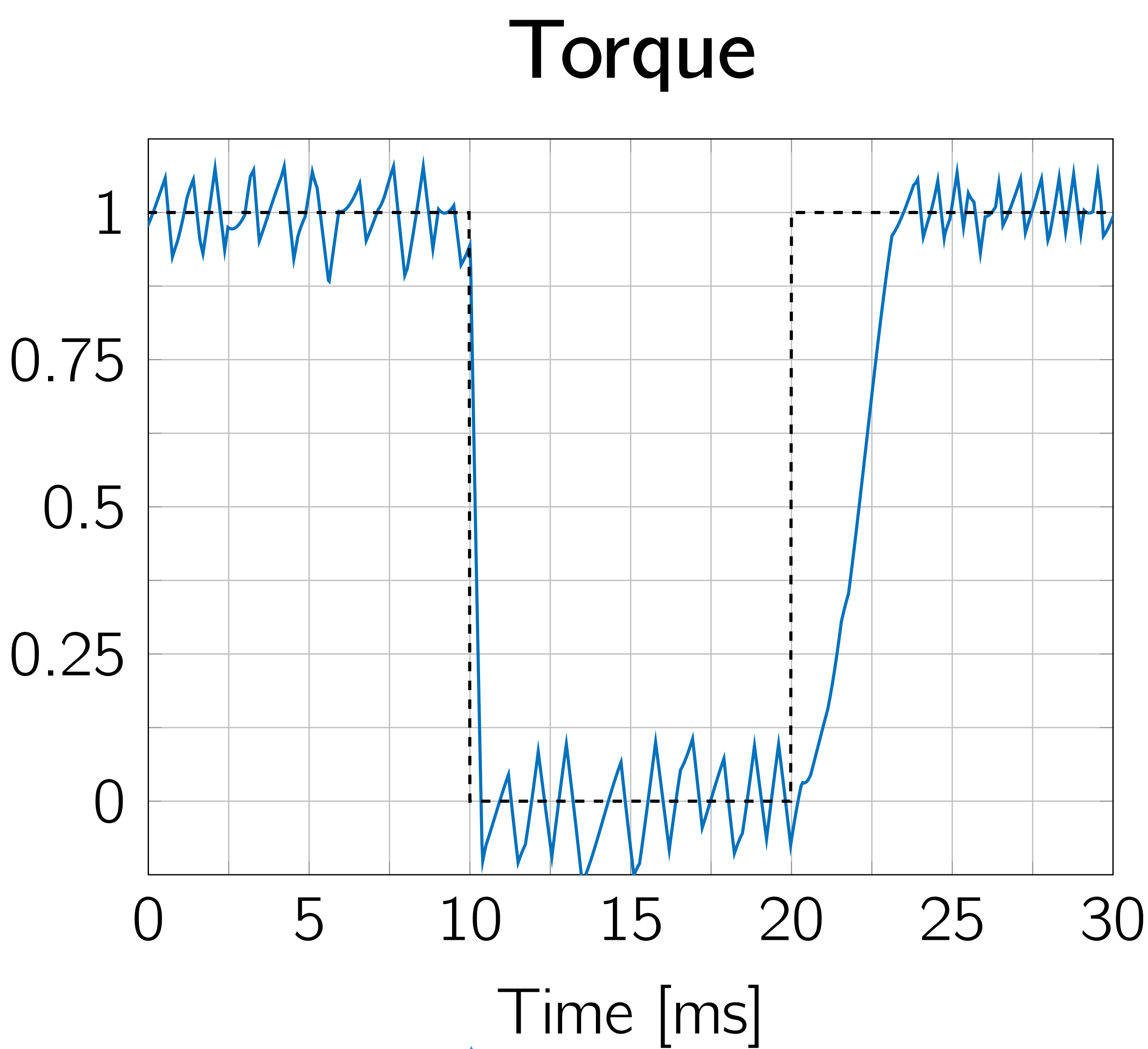
Torque

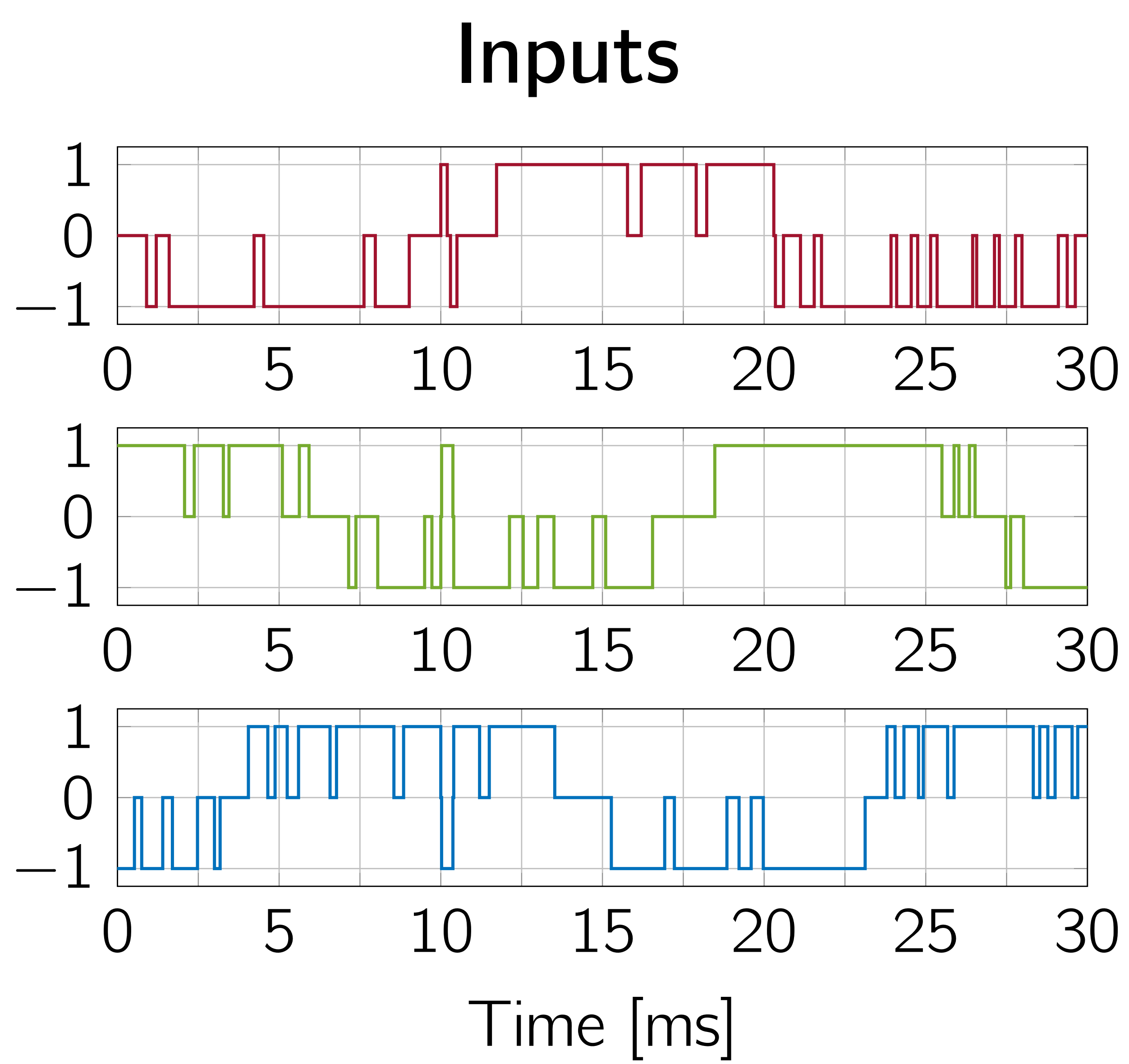


Inputs



Transients

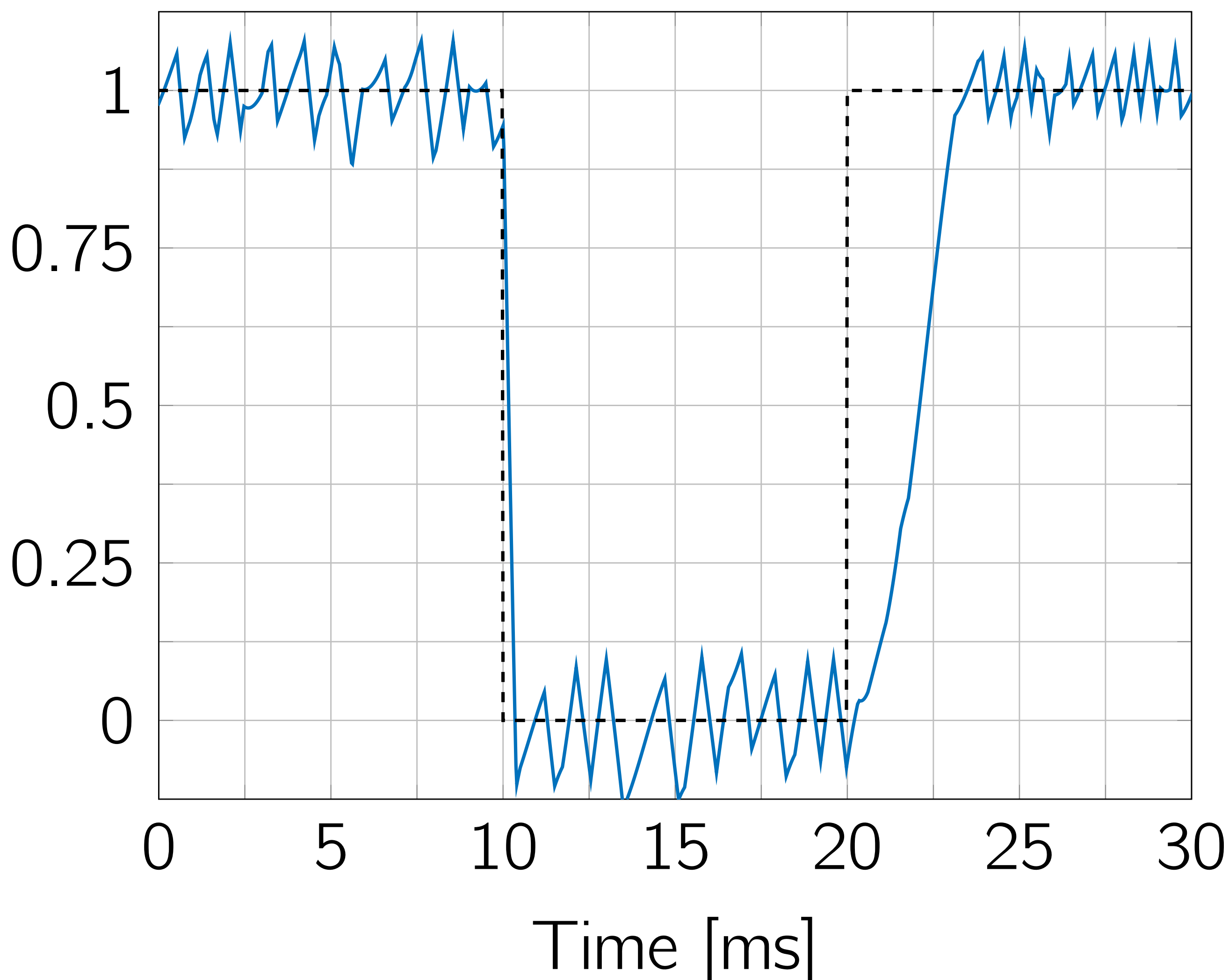




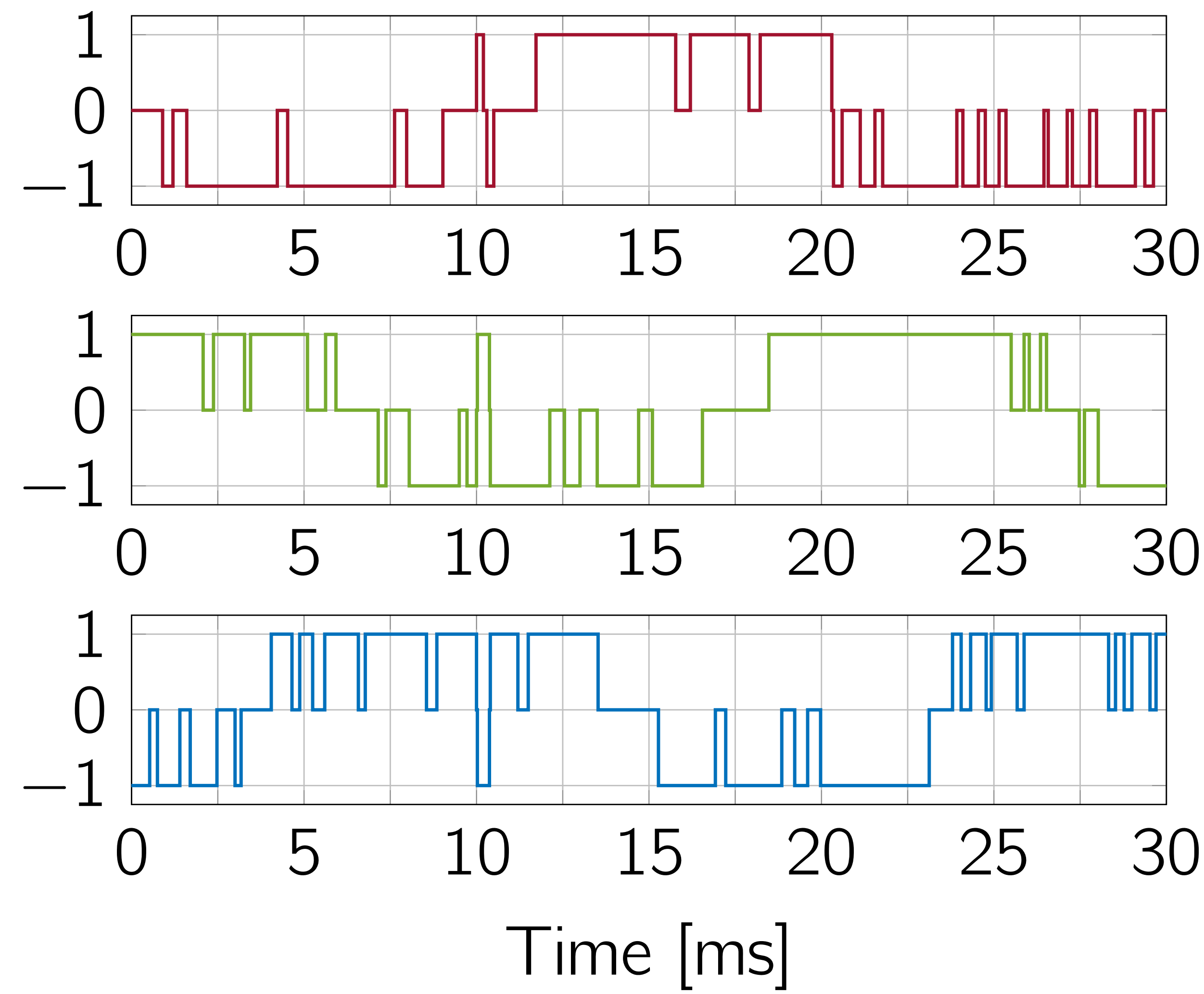
0.35 ms

Transients

Torque

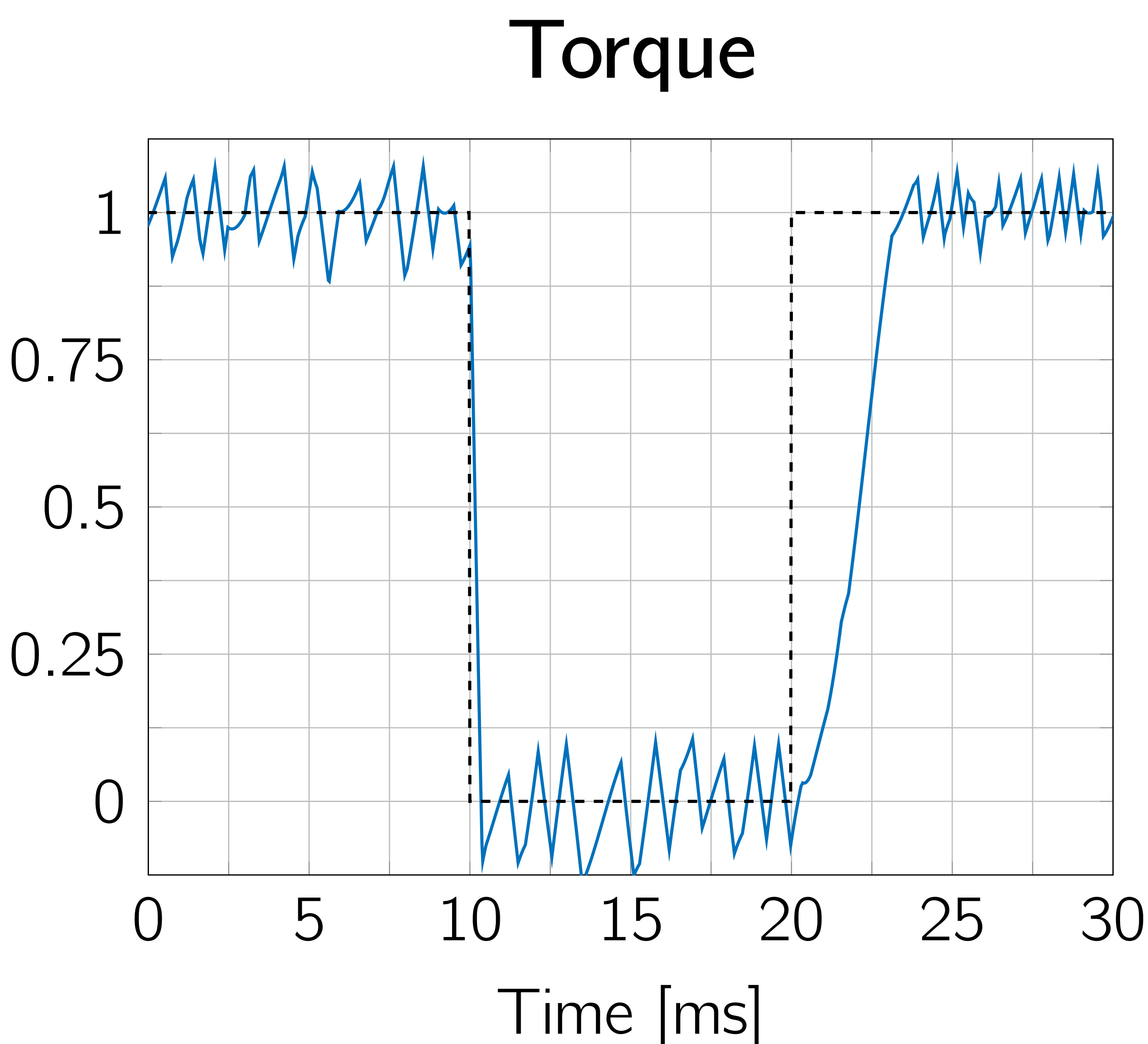


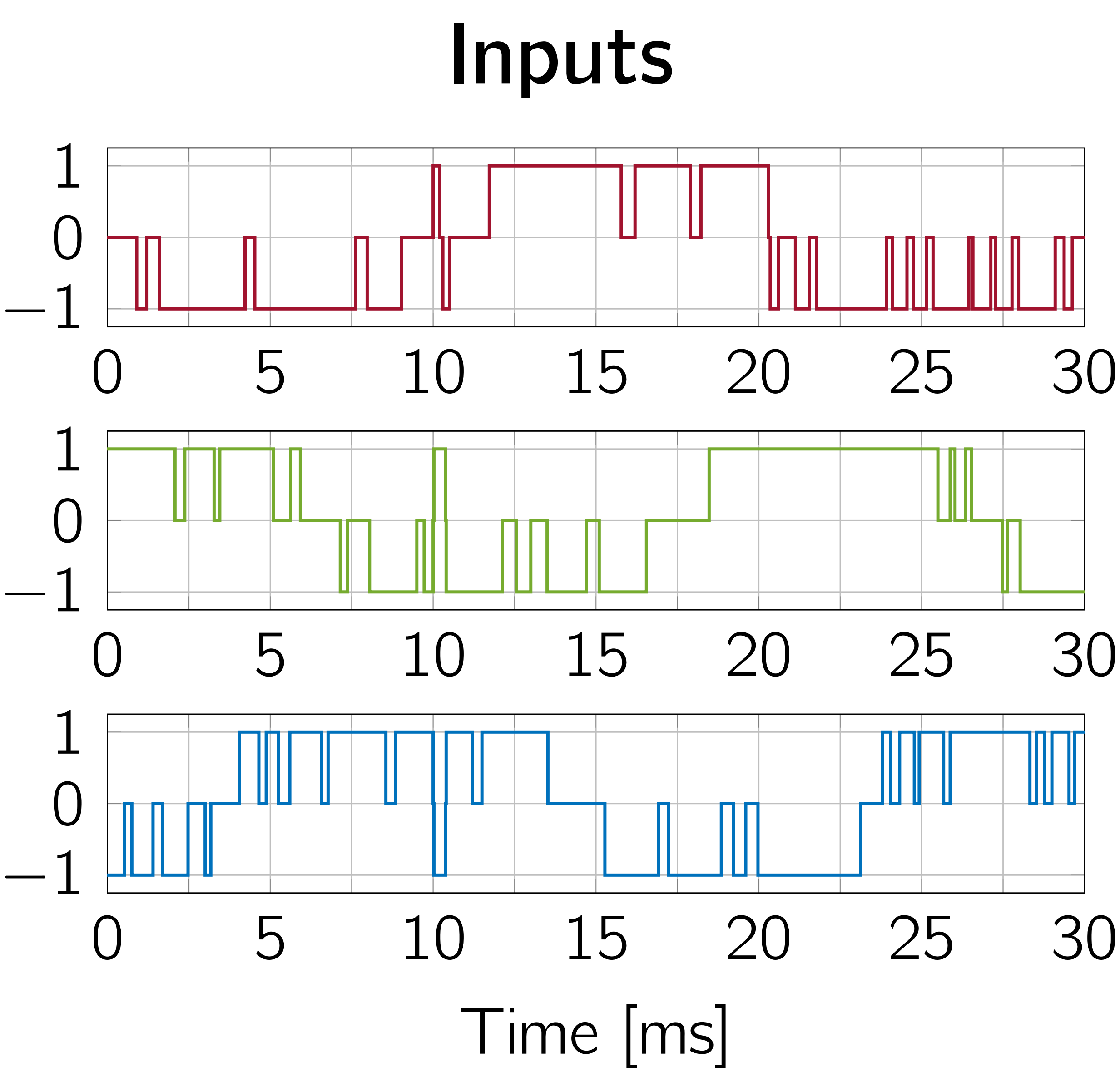
Inputs



0.35 ms

3.5 ms





0.35 ms

3.5 ms

Very Fast Transient Times!

Conclusions - New Approach to Direct MPC

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- Meaningful formulation (THD vs f_{sw})

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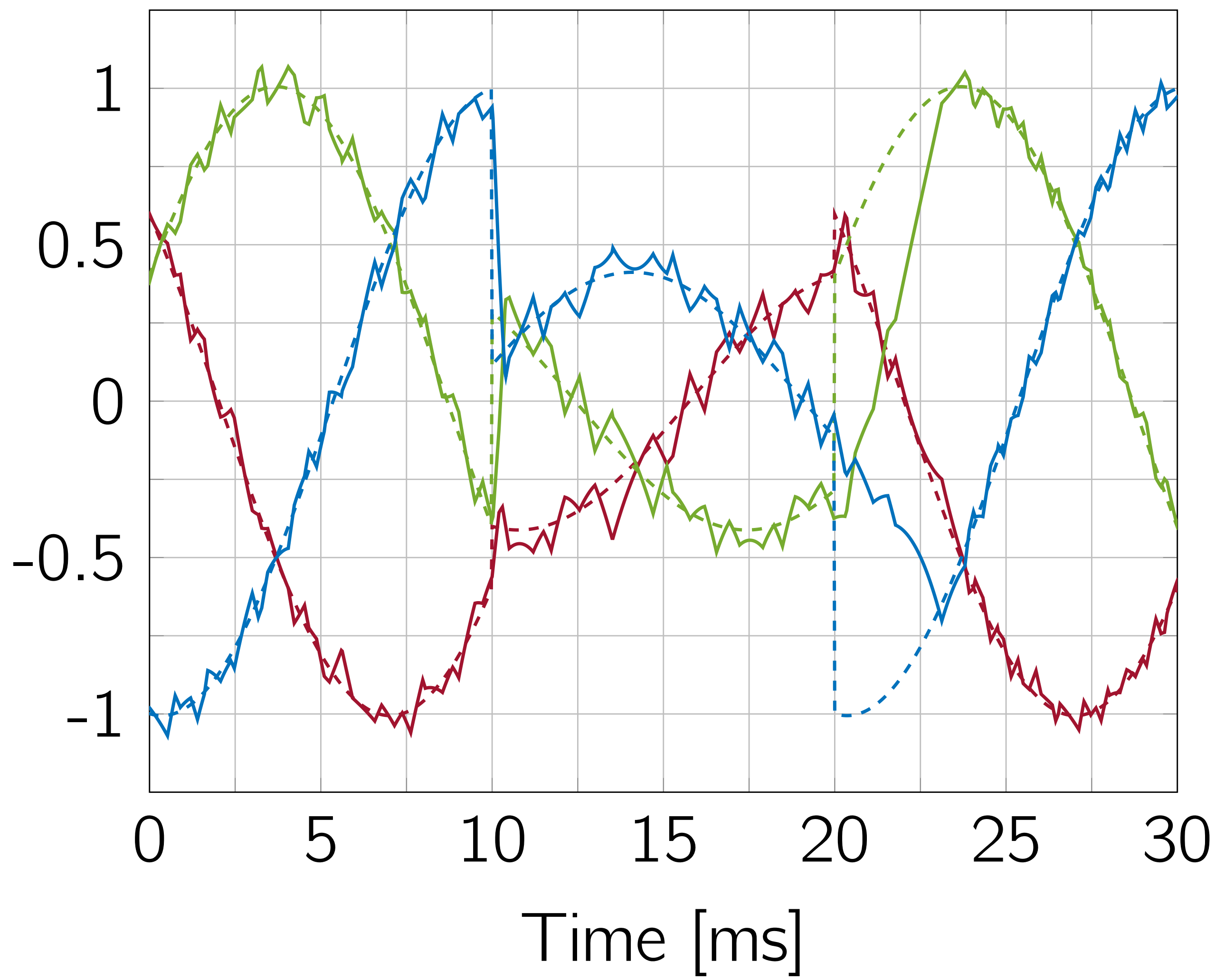
"High-Speed Finite Control Set Model Predictive Control for Power Electronics"
B. Stellato, T. Geyer, P. J. Goulart
IEEE Transactions on Power Electronics (In Press)

$$T = C(\Psi_r \times i_s)$$



Integer Program

$$
\begin{array}{ll}
\text{minimize} & \frac{1}{2}U^\top QU + f(x_0)^\top U \\
\text{subject to} & A_{ineq}U \leq b_{ineq}(x_0) \\
& U \in \{-1, 0, 1\}^{3N}
\end{array}
$$

Quadratic Cost Function

$$\int_{\mathcal{X}} V_0(z)c(\mathrm{d}z) = \mathrm{Tr}(P_0\Sigma_c) + 2q_0^\top \mu_c + r_0$$

SDP Formulation

$$\begin{array}{ll}
\text{maximize} & \mathrm{Tr}\left(P_0\Sigma_c\right) + 2q_0^\top \mu_c + r_0 \\
\text{subject to} & \tilde{M}_i(m) \succeq 0, \quad \forall m \in \mathcal{M}, \quad i = 1, \ldots, M \\
 & V_0 = V_M \\
 & P_i \in \mathbb{S}^{n_x},\ q_i \in \mathbb{R}^{n_x},\ r_i \in \mathbb{R},\ i = 0, \ldots, M
\end{array}$$

Backup - Switch Positions