



High-Speed Integer Optimal Control using ADP

Bartolomeo Stellato and Paul Goulart

EUCCO 2016

13/09/2016 Leuven, Belgium

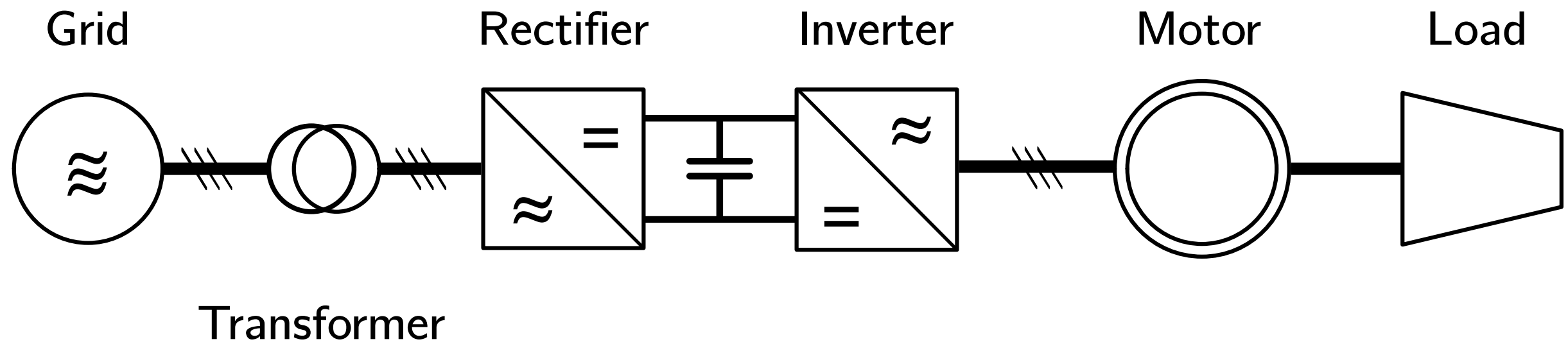


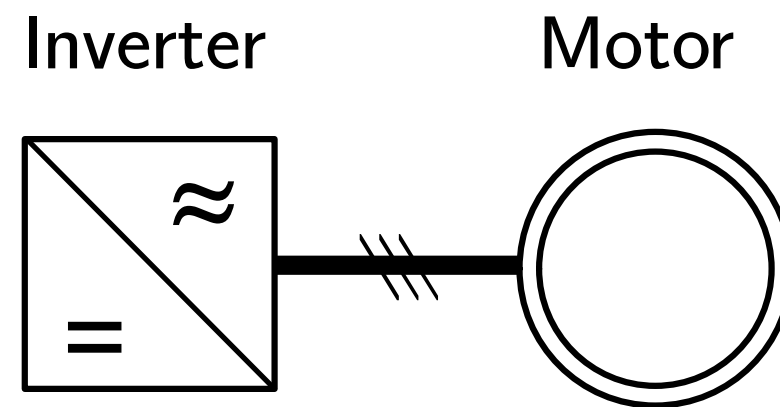
UNIVERSITY OF
OXFORD



**Medium-voltage
drives market is
worth 4bln \$....**

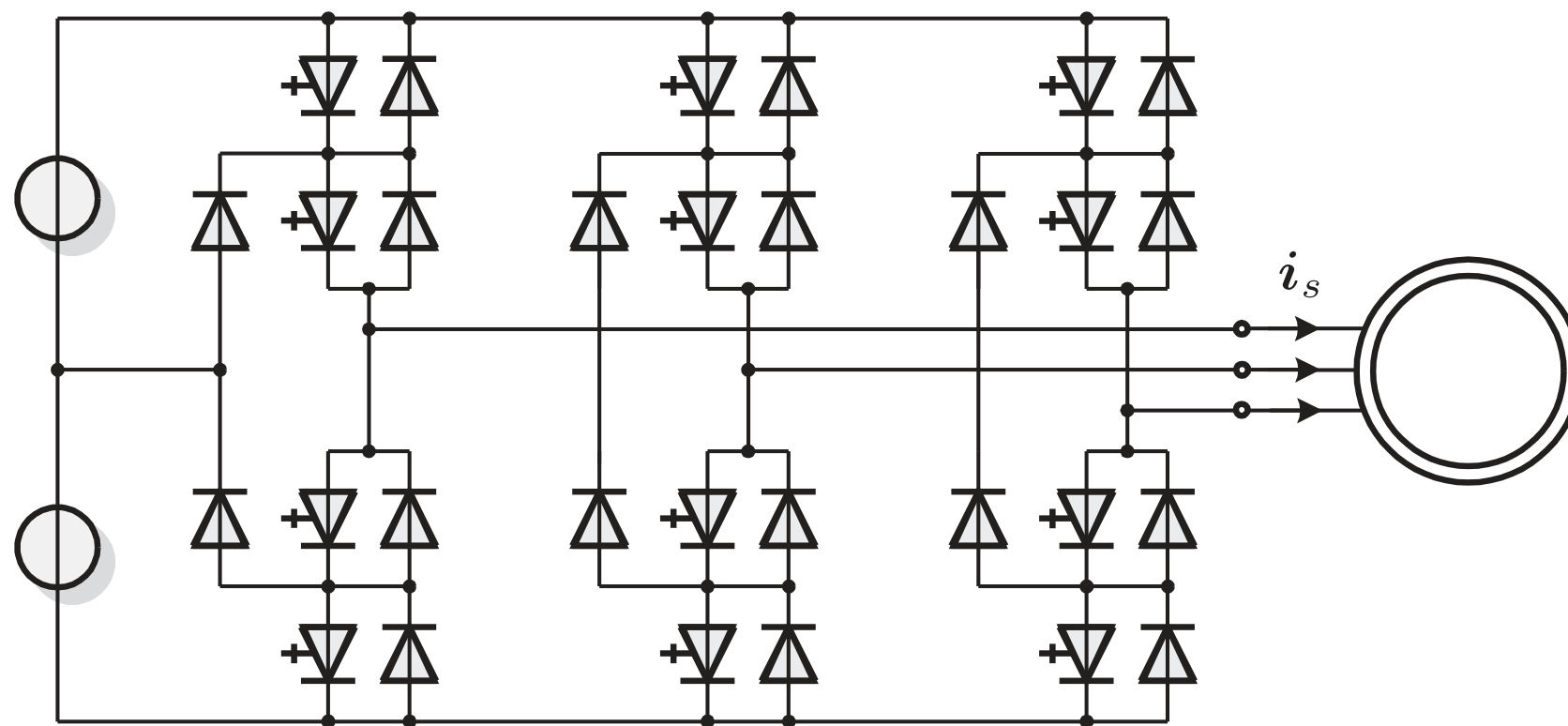
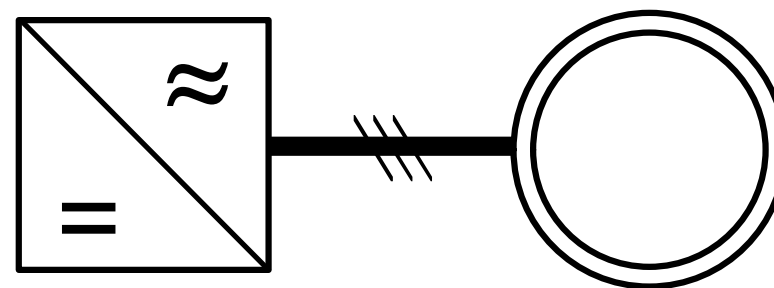
**... and is growing
10% every year!**



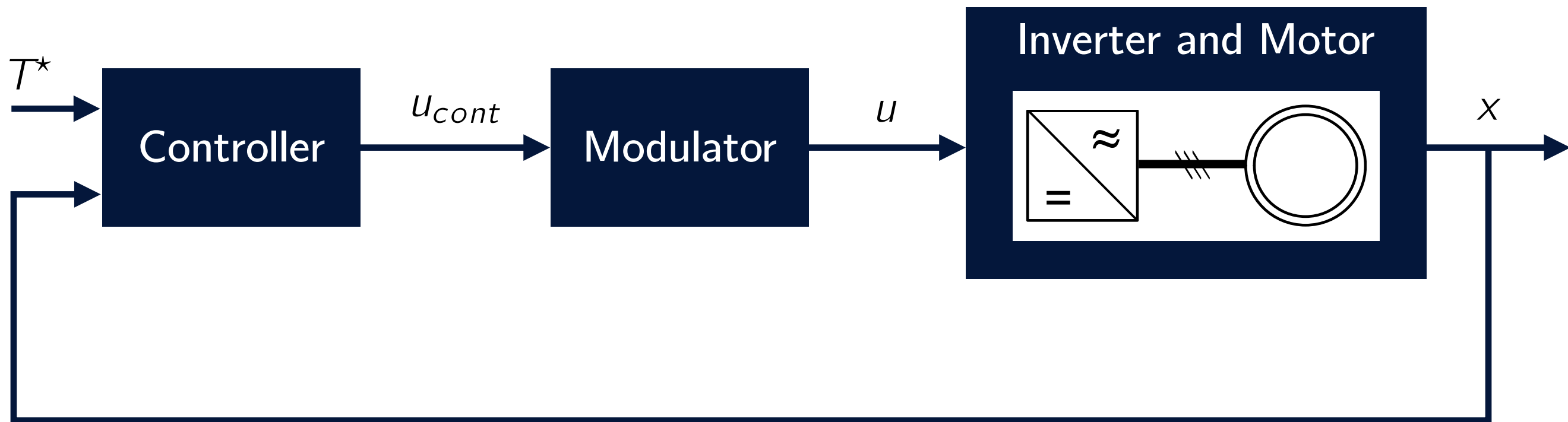


Inverter

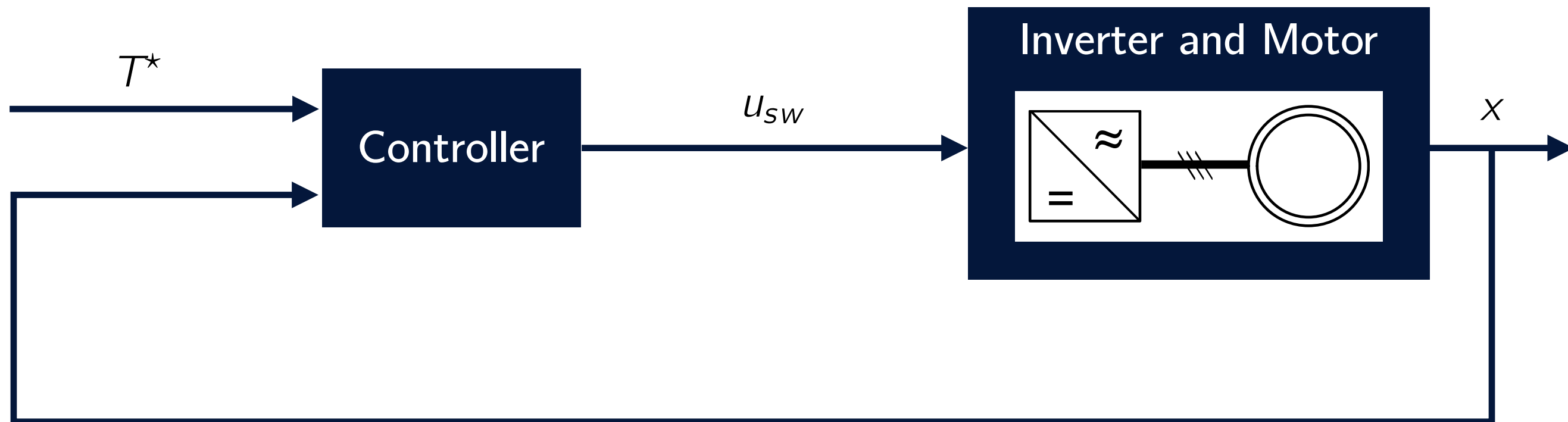
Motor



Traditional Control

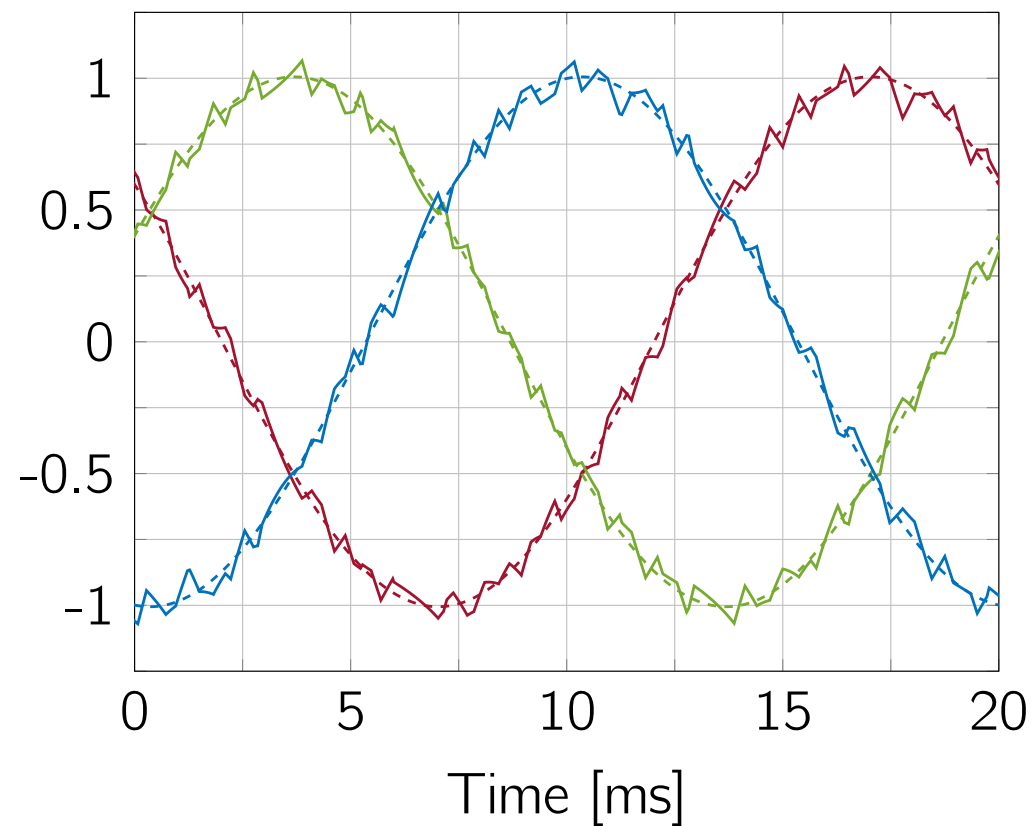


Direct MPC

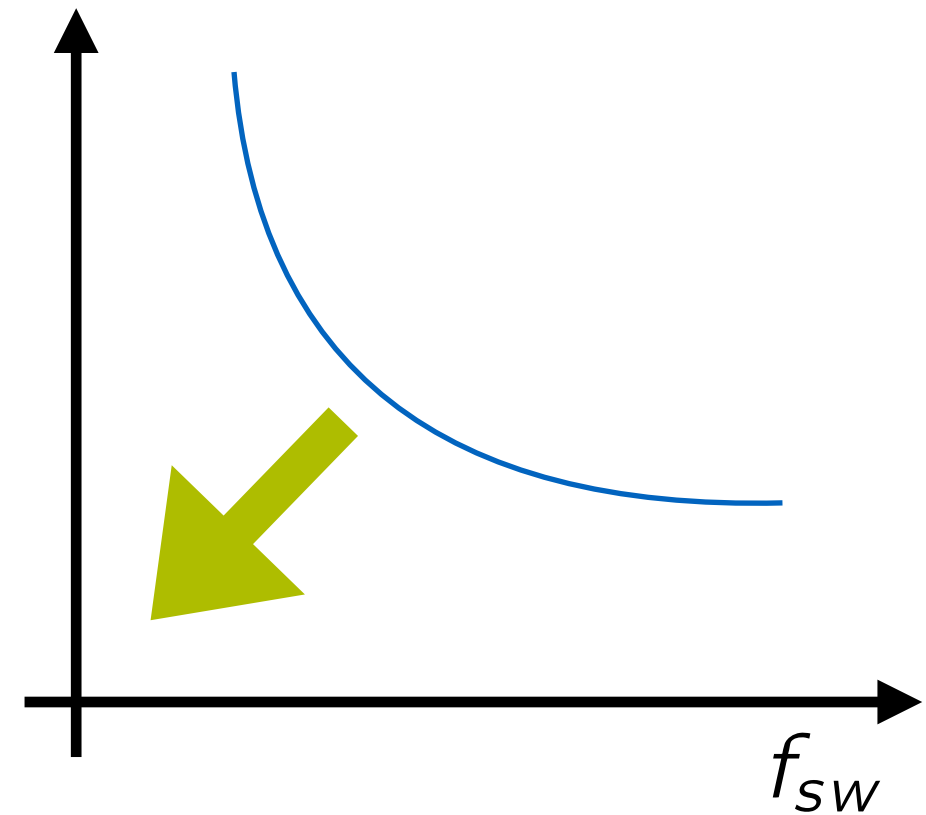


... But Direct MPC is Difficult

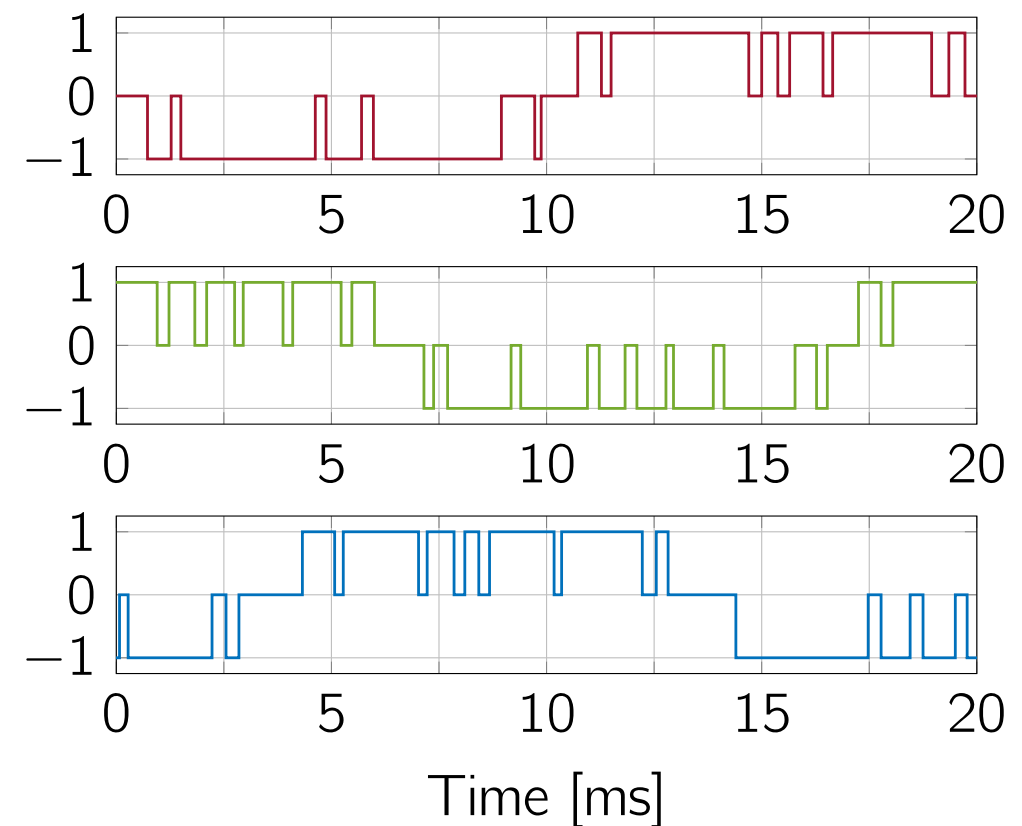
Currents



THD

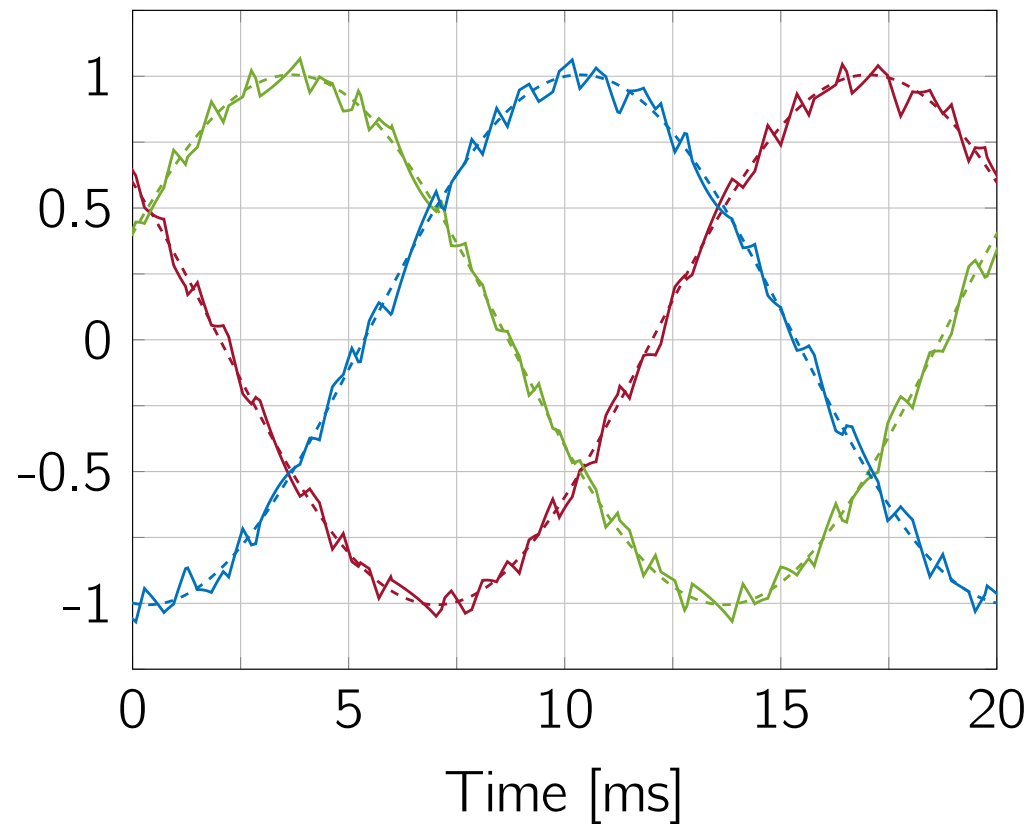


Inputs

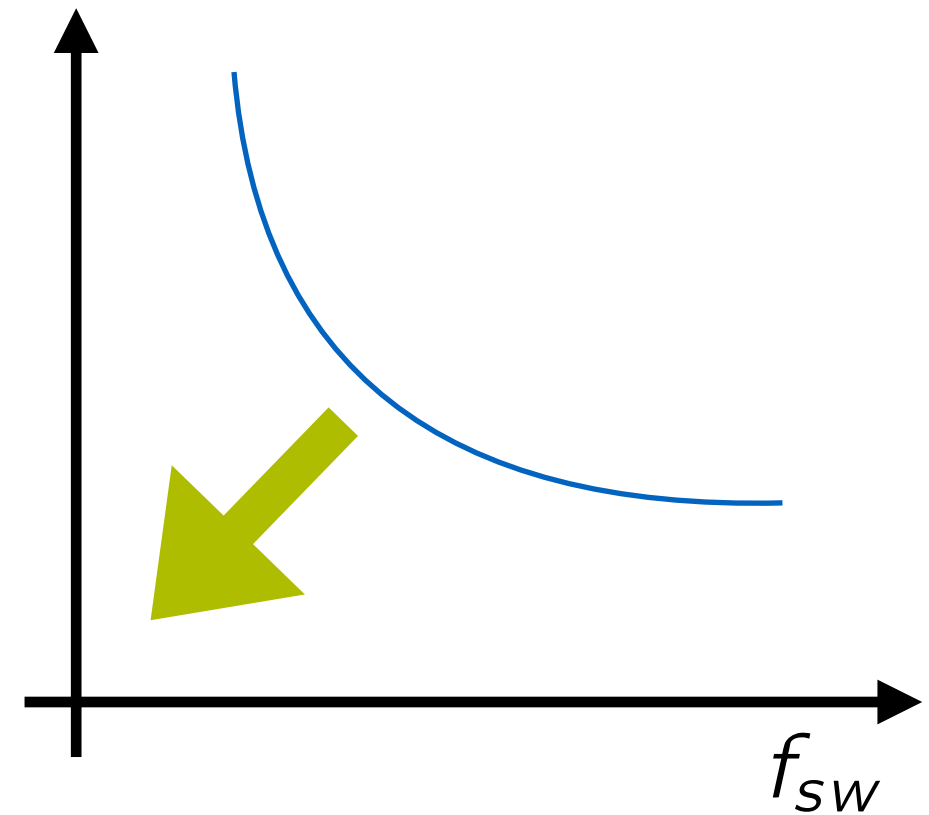


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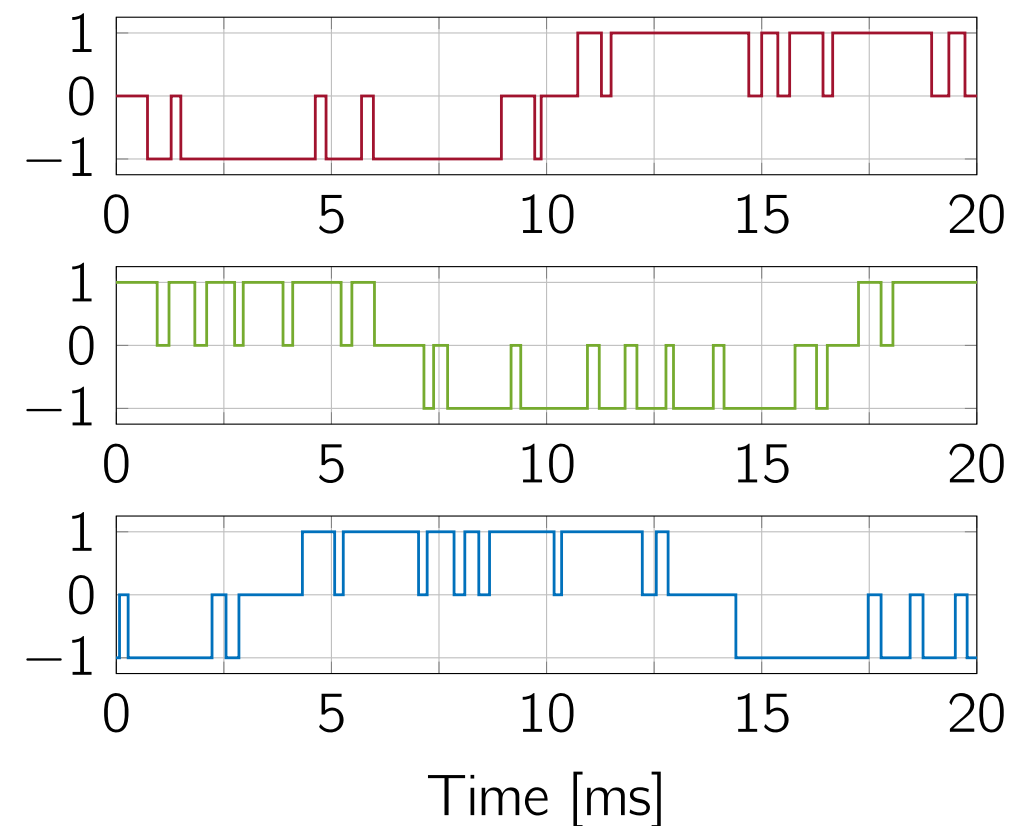
Currents



THD



Inputs



Tradeoff
 THD vs f_{sw}

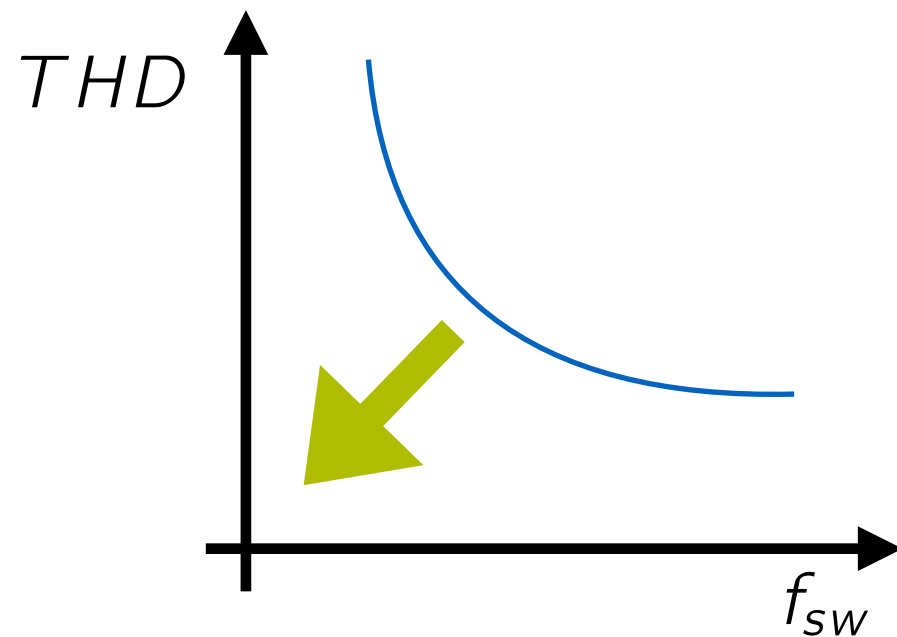
Very fast dynamics

Very fast dynamics



Mixed Integer
Optimization
Problems in $25\mu s$!

Control Objectives



Tradeoff
 THD vs f_{sw}

Timing



Mixed Integer
Optimization
Problems in $25\mu s$!

Problem Formulation

$$THD \sim \sqrt{\frac{1}{M} \sum_{k=0}^{\infty} \|i(k) - i^*(k)\|_2^2}$$

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THD Component in Cost Function

$$\sum_{k=0}^{\infty} \gamma^k \|i(k) - i^*(k)\|_2^2$$

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Internal Motor States
 x_m

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Internal Motor States
 x_m

Oscillating References
 x_{osc}

$$f_{sw} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{k=0}^T \|u(k) - u(k-1)\|_1$$

$$f_{sw} = \lim_{T \rightarrow \infty} \underbrace{\frac{1}{T} \sum_{k=0}^T \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

FIR Filter

$$f_{sw} = \lim_{T \rightarrow \infty} \underbrace{\frac{1}{T} \sum_{k=0}^T \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$



Approximate
with IIR Filter

$$f_{sw} = \lim_{T \rightarrow \infty} \underbrace{\frac{1}{T} \sum_{k=0}^T \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}} \longrightarrow \text{Approximate with IIR Filter}$$

f_{sw} Component in Cost Function

$$\delta \sum_{k=0}^{\infty} \gamma^k \|\hat{f}_{sw}(k) - f_{sw}^*\|_2^2$$

$$f_{sw} = \lim_{T \rightarrow \infty} \underbrace{\frac{1}{T} \sum_{k=0}^T \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$



Approximate
with IIR Filter

f_{sw} Component in Cost Function

$$\delta \sum_{k=0}^{\infty} \gamma^k \|\hat{f}_{sw}(k) - f_{sw}^*\|_2^2$$

Frequency
Estimate



$$f_{sw} = \lim_{T \rightarrow \infty} \underbrace{\frac{1}{T} \sum_{k=0}^T \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

Approximate
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f_{sw} Component in Cost Function

$$\delta \sum_{k=0}^{\infty} \gamma^k \|\hat{f}_{sw}(k) - f_{sw}^*\|_2^2$$

Frequency
Estimate

Desired
Frequency

$$f_{sw} = \lim_{T \rightarrow \infty} \underbrace{\frac{1}{T} \sum_{k=0}^T \|u(k) - u(k-1)\|_1}_{\text{FIR Filter}}$$

Approximate
with IIR Filter

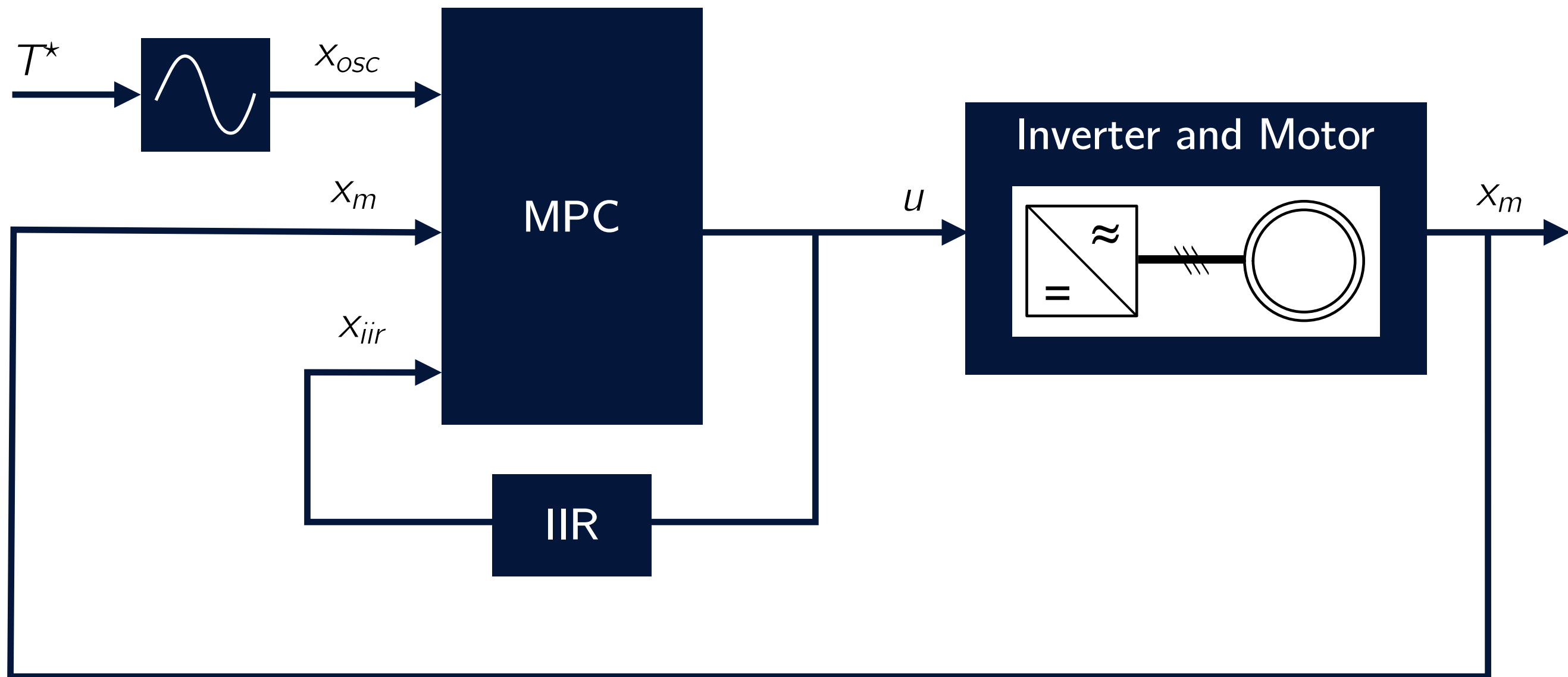
f_{sw} Component in Cost Function

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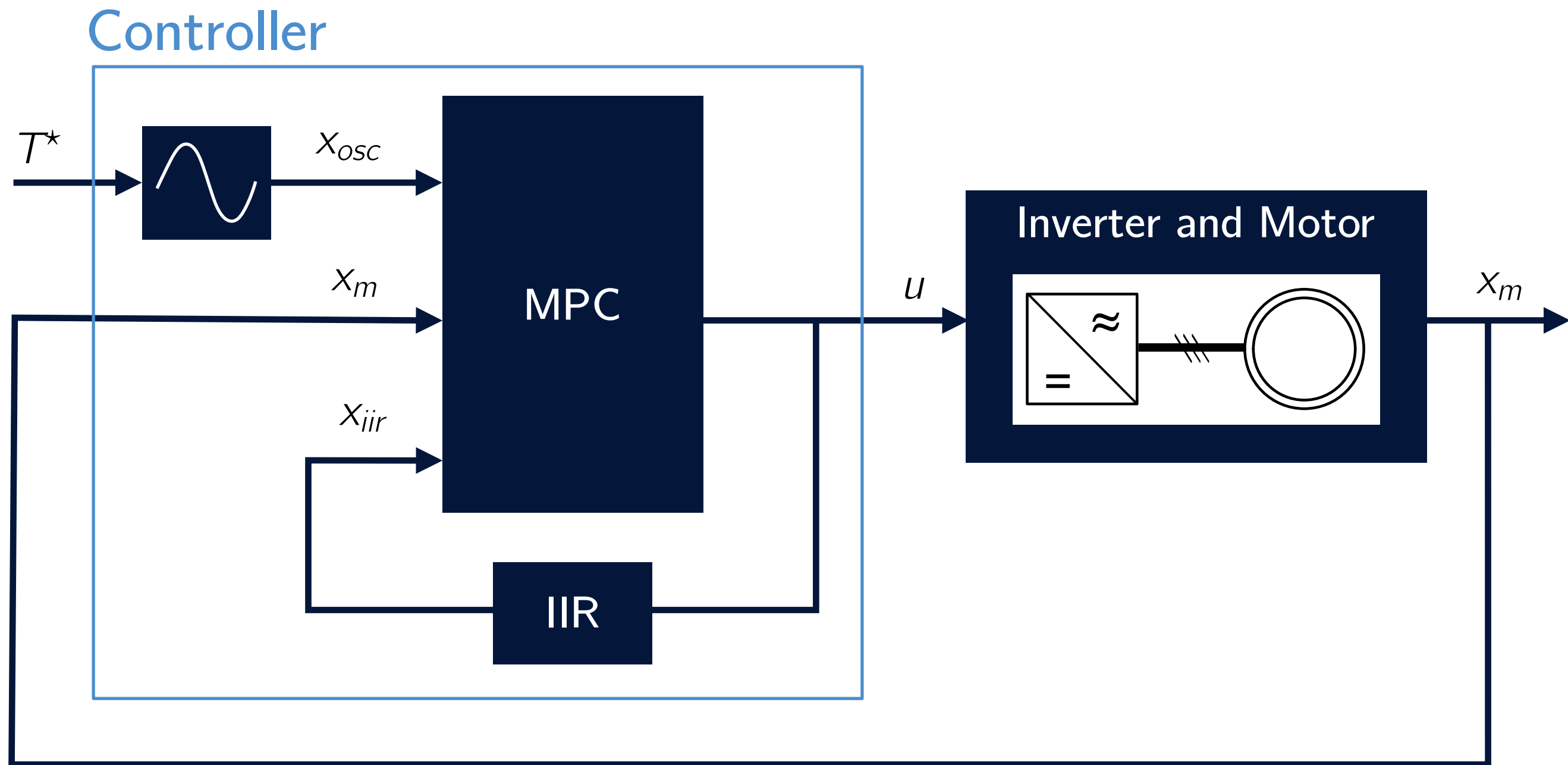
Frequency
Estimate

Desired
Frequency

Filter States
 x_{iir}



$$\text{States: } x = \begin{bmatrix} x_{osc}^T & x_{im}^T & x_{iir}^T \end{bmatrix}^T$$



$$\text{States: } x = \begin{bmatrix} x_{osc}^T & x_{im}^T & x_{iir}^T \end{bmatrix}^T$$

Infinite Horizon

$$\begin{aligned} &\text{minimize} && \sum_{k=0}^{\infty} \gamma^k l(x(k)) \\ &\text{subject to} && x(k+1) = Ax(k) + Bu(k) \\ & && x(0) = x_0 \\ & && x(k) \in \mathcal{X}, u(k) \in \{-1, 0, 1\}^3 \end{aligned}$$

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$$\text{where} \quad l(x(k)) = \underbrace{\|i(k) - i^*(k)\|_2^2}_{THD} + \delta \underbrace{\|\hat{f}_{sw}(k) - f_{sw}^*\|_2^2}_{f_{sw}}$$

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Impossible to Solve Online

Problem Solution

Infinite Horizon

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Short Finite Horizon

$$\begin{aligned} &\text{minimize} && \sum_{k=0}^{N-1} \gamma^k l(x(k)) + \gamma^N V(x(k)) \\ &\text{subject to} && x(k+1) = Ax(k) + Bu(k) \\ &&& x(0) = x_0 \\ &&& x(k) \in \mathcal{X}, u(k) \in \{-1, 0, 1\}^3 \end{aligned}$$

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Very Short
Horizon N

Short Finite Horizon

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Very Short
Horizon N

Approximate
 $V(x(k))$
Offline

Bellman Equality

$$V^*(x) = \min_u \{l(x) + \gamma V^*(Ax + Bu)\}$$

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$\mathcal{T}V^*$

Bellman Operator $\mathcal{T}$

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Bellman Operator $\mathcal{T}$

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Monotone $V_1 \leq V_2 \implies \mathcal{T}V_1 \leq \mathcal{T}V_2$

Bellman Inequality

$$V(x) \leq \underbrace{\min_u \{l(x) + \gamma V(Ax + Bu)\}}_{\mathcal{T}V}$$

Bellman Operator $\mathcal{T}$

Contractive

$$\lim_{M \rightarrow \infty} \mathcal{T}^M V = V^*$$

Monotone

$$V_1 \leq V_2 \implies \mathcal{T}V_1 \leq \mathcal{T}V_2$$

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Bellman Operator $\mathcal{T}$

Contractive $\lim_{M \rightarrow \infty} \mathcal{T}^M V = V^*$

Monotone $V_1 \leq V_2 \implies \mathcal{T}V_1 \leq \mathcal{T}V_2$

Sufficient condition for underestimating V^*

$$V \leq \mathcal{T}V \implies V \leq V^*$$

Infinite Dimensional LP

$$\begin{array}{ll}\text{maximize} & \int_{\mathcal{X}} V(x) c(dx) \\ \text{subject to} & V(x) \leq \mathcal{T}V(x) \quad \forall x\end{array}$$

Infinite Dimensional LP

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Iterated Bellman Inequality

Infinite Dimensional LP

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Iterated Bellman Inequality

Restrict to Quadratic Functions

$$V(x) = x^\top P x + 2q^\top x + r$$

Infinite Dimensional LP

$$\begin{array}{ll} \text{maximize} & \int_{\mathcal{X}} V(x) c(dx) \\ \text{subject to} & V(x) \leq \mathcal{T}^M V(x) \quad \forall x \end{array}$$

Iterated Bellman Inequality



Restrict to Quadratic
Functions

$$V(x) = x^\top P x + 2q^\top x + r$$



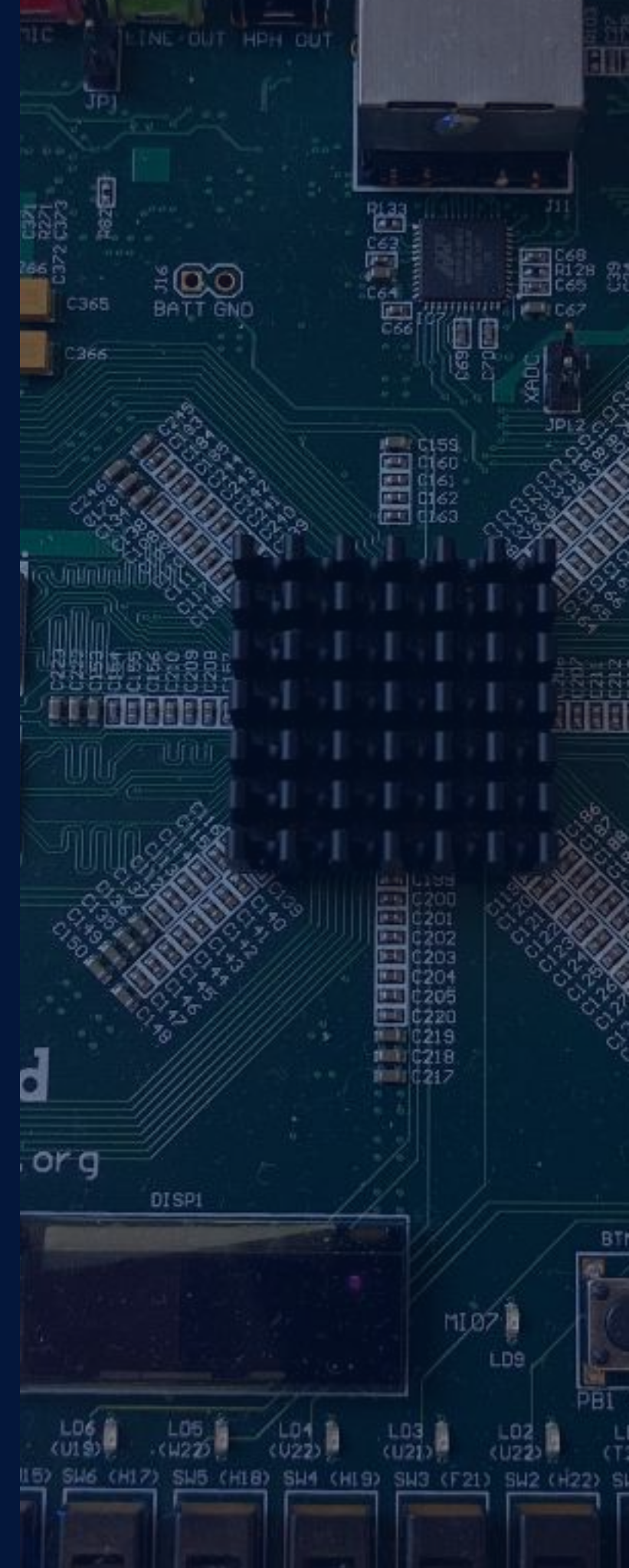
Tractable
SDP
(Offline)

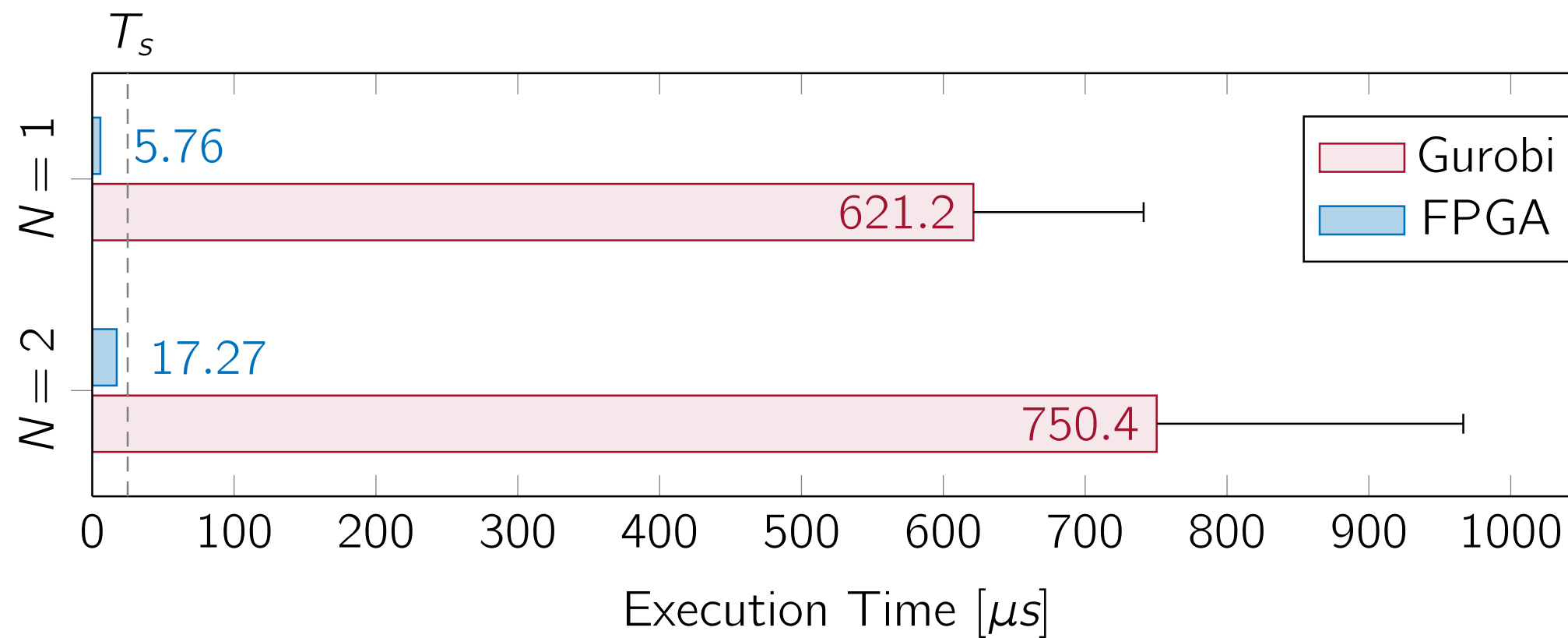
FPGA Implementation

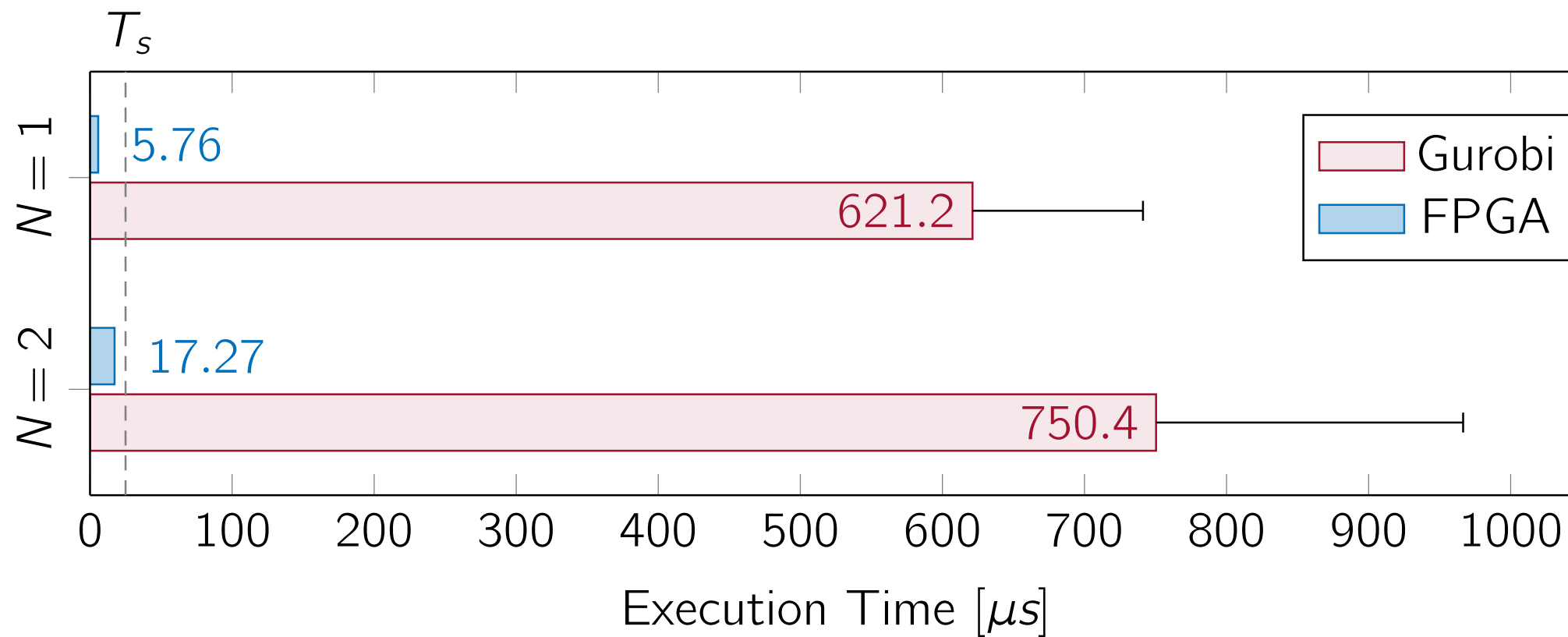
PROTOIP Toolbox

(Suardi et al.)

- Brute force enumeration
- Pipelined Evaluation of Switch Sequences
- Parallelized Matrix Computations

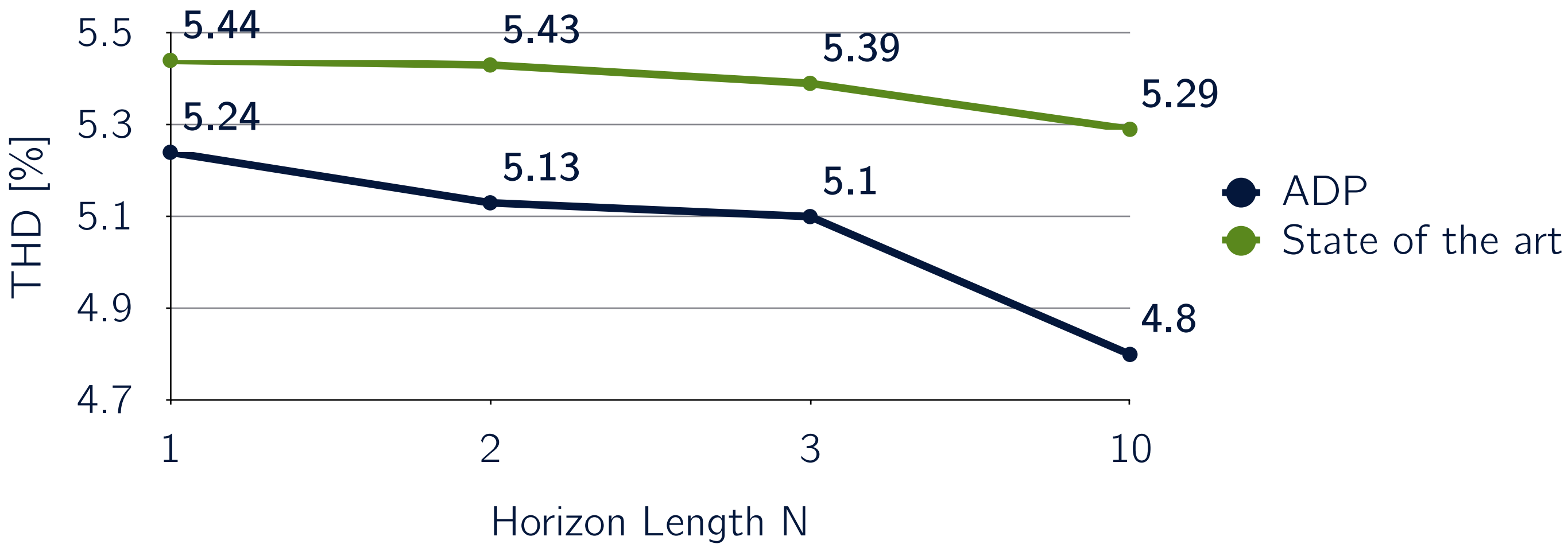


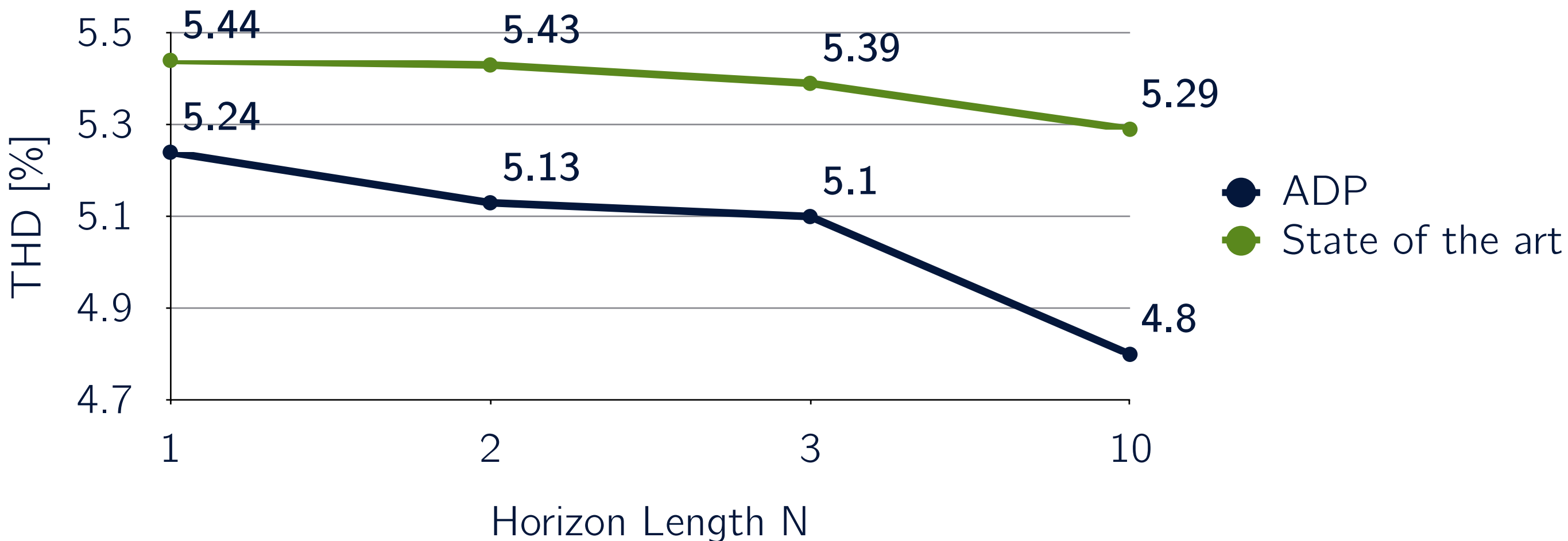




Computation Times under $25 \mu s$!

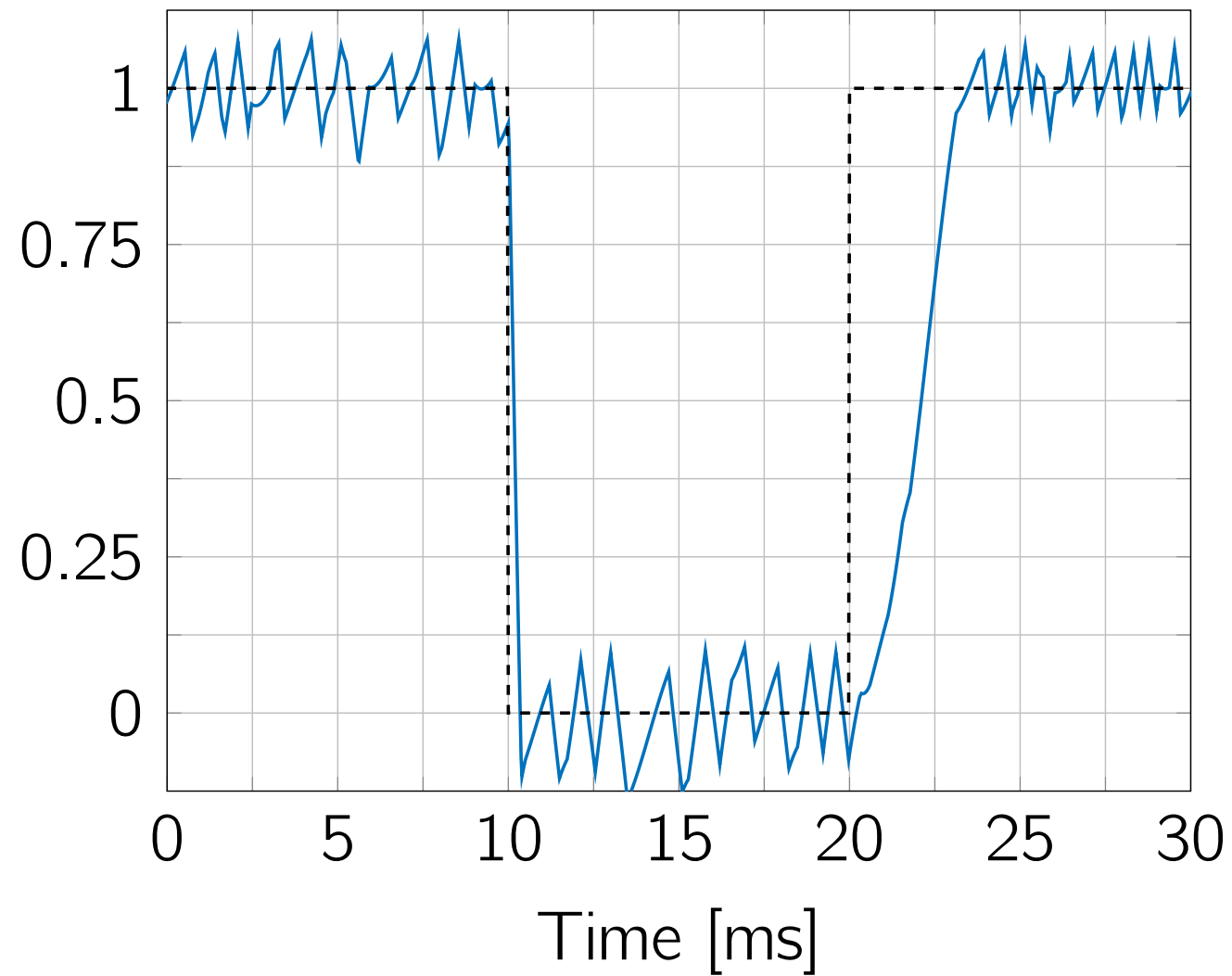
Hardware in the Loop Tests



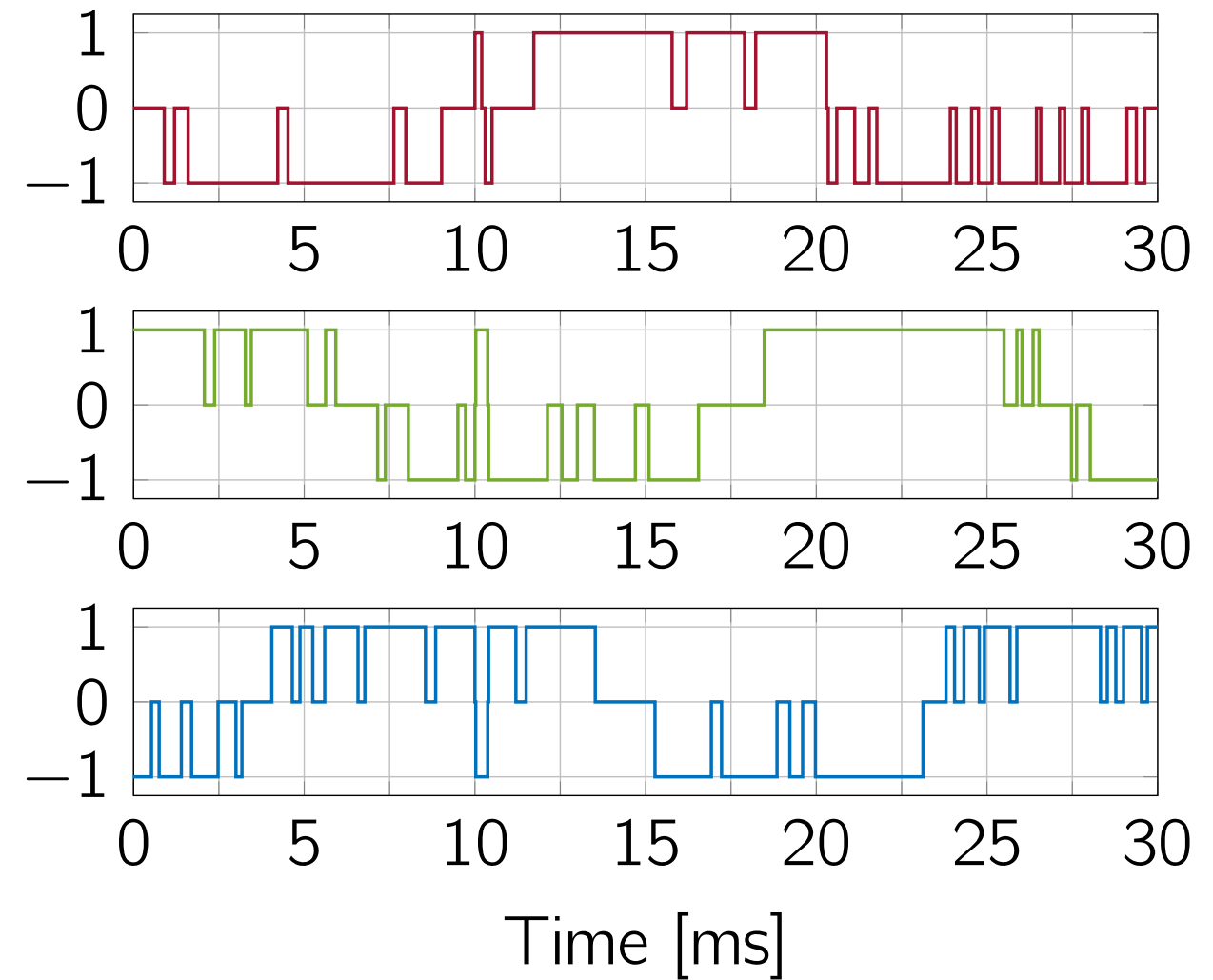


Better Performance with Short Horizons

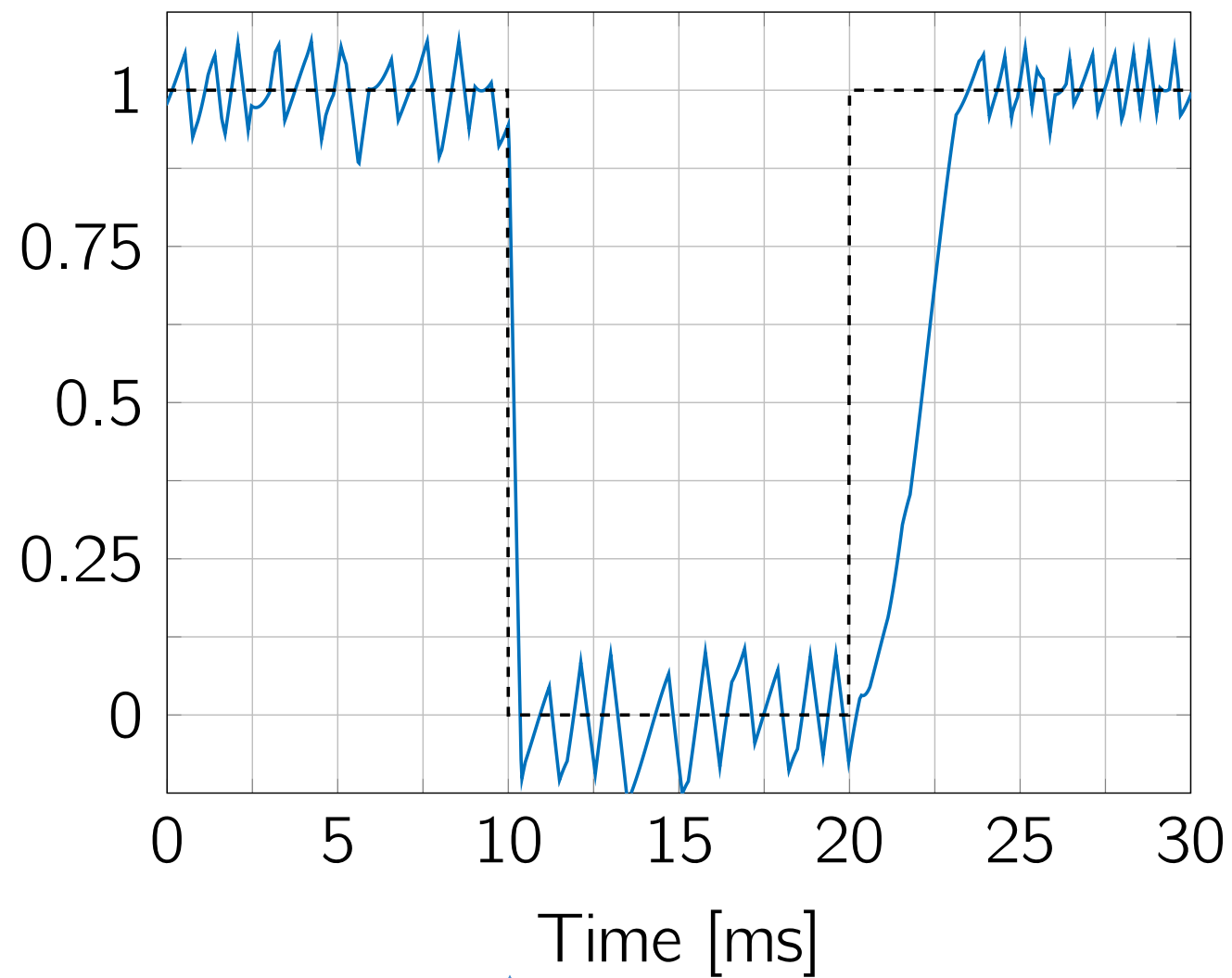
Torque



Inputs

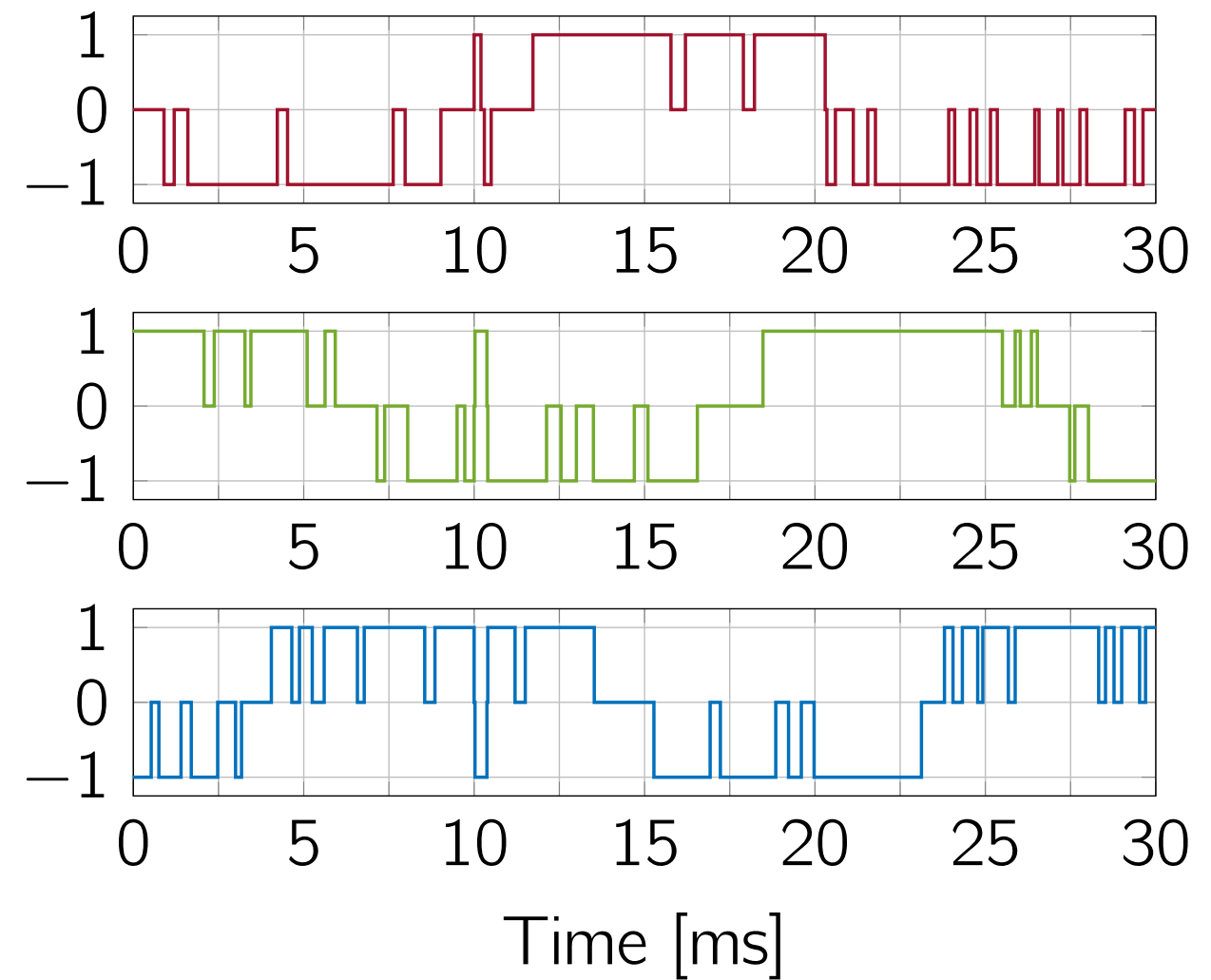


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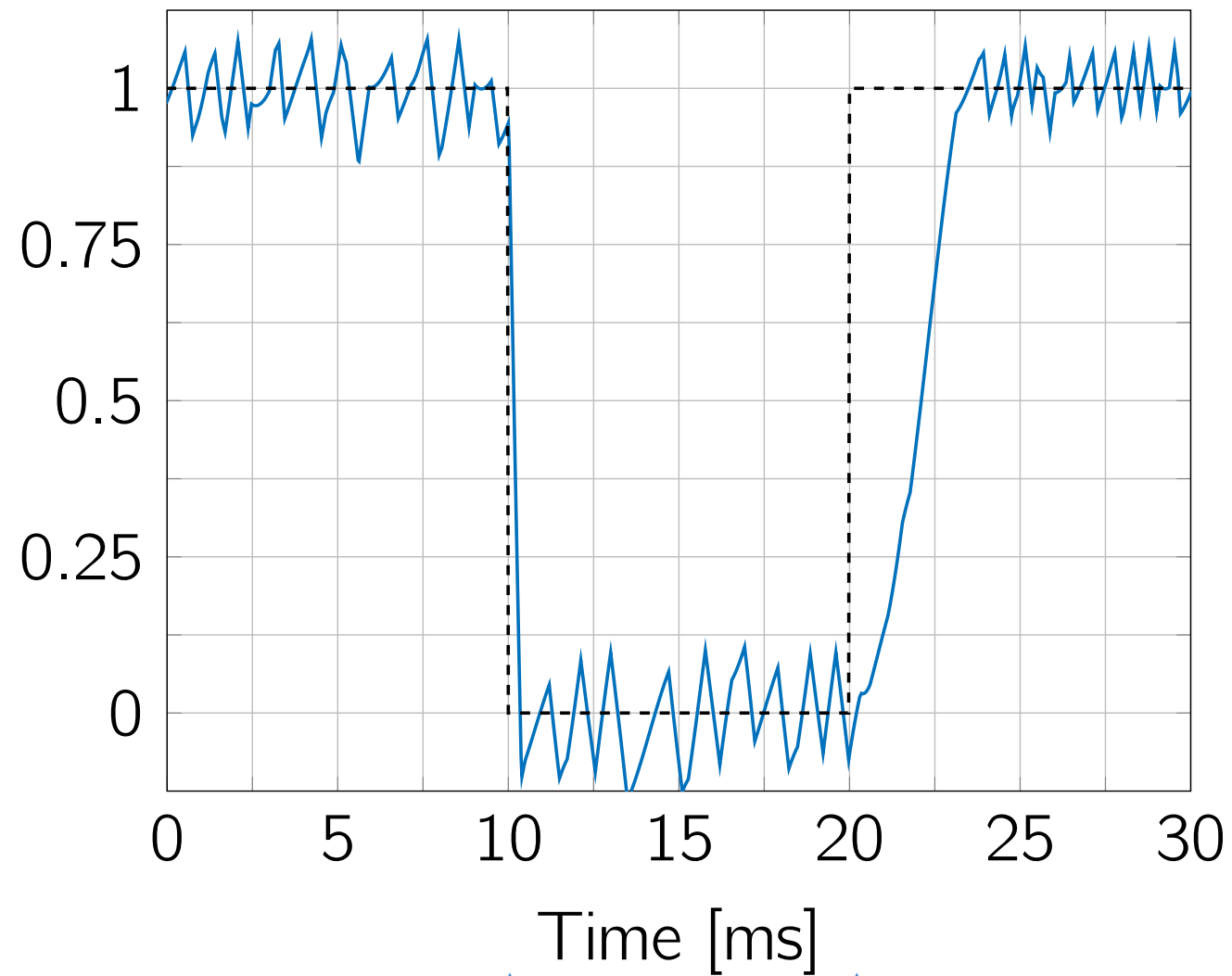


0.35 ms

Inputs



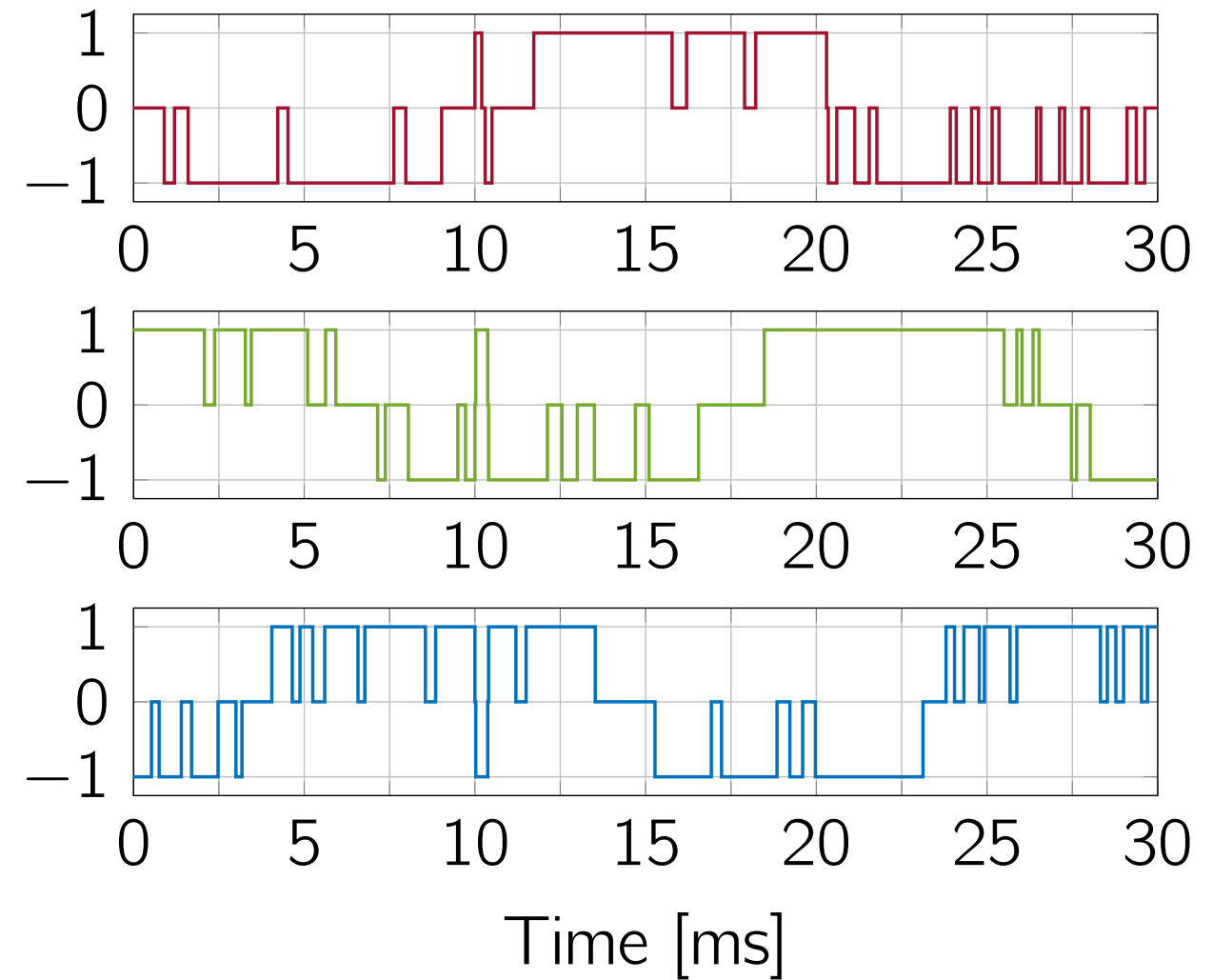
Torque



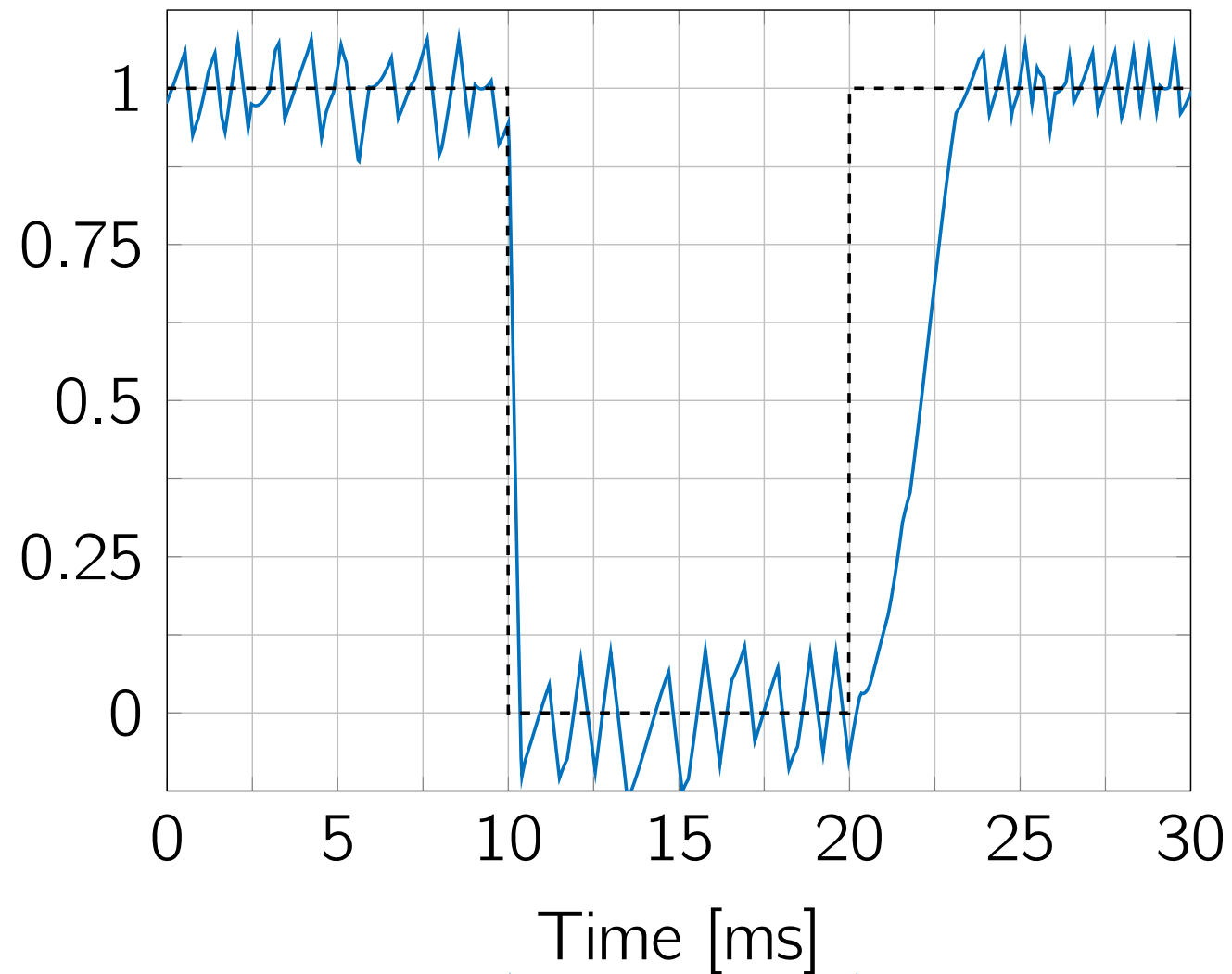
0.35 ms

3.5 ms

Inputs



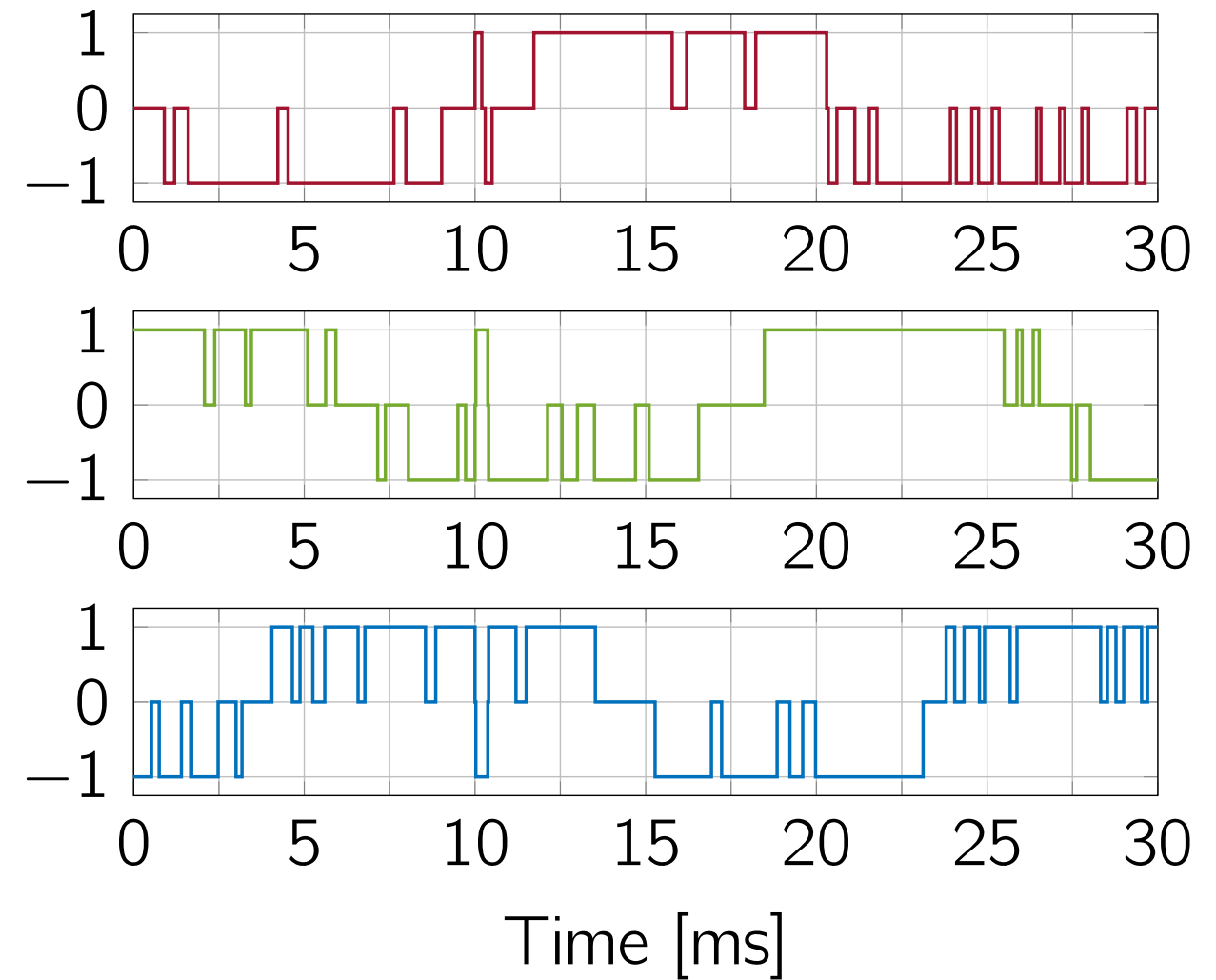
Torque



0.35 ms

3.5 ms

Inputs



**Very Fast
Transient
Times!**

- Meaningful formulation (THD vs f_{sw})

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- Complexity reduction with good performance (ADP)

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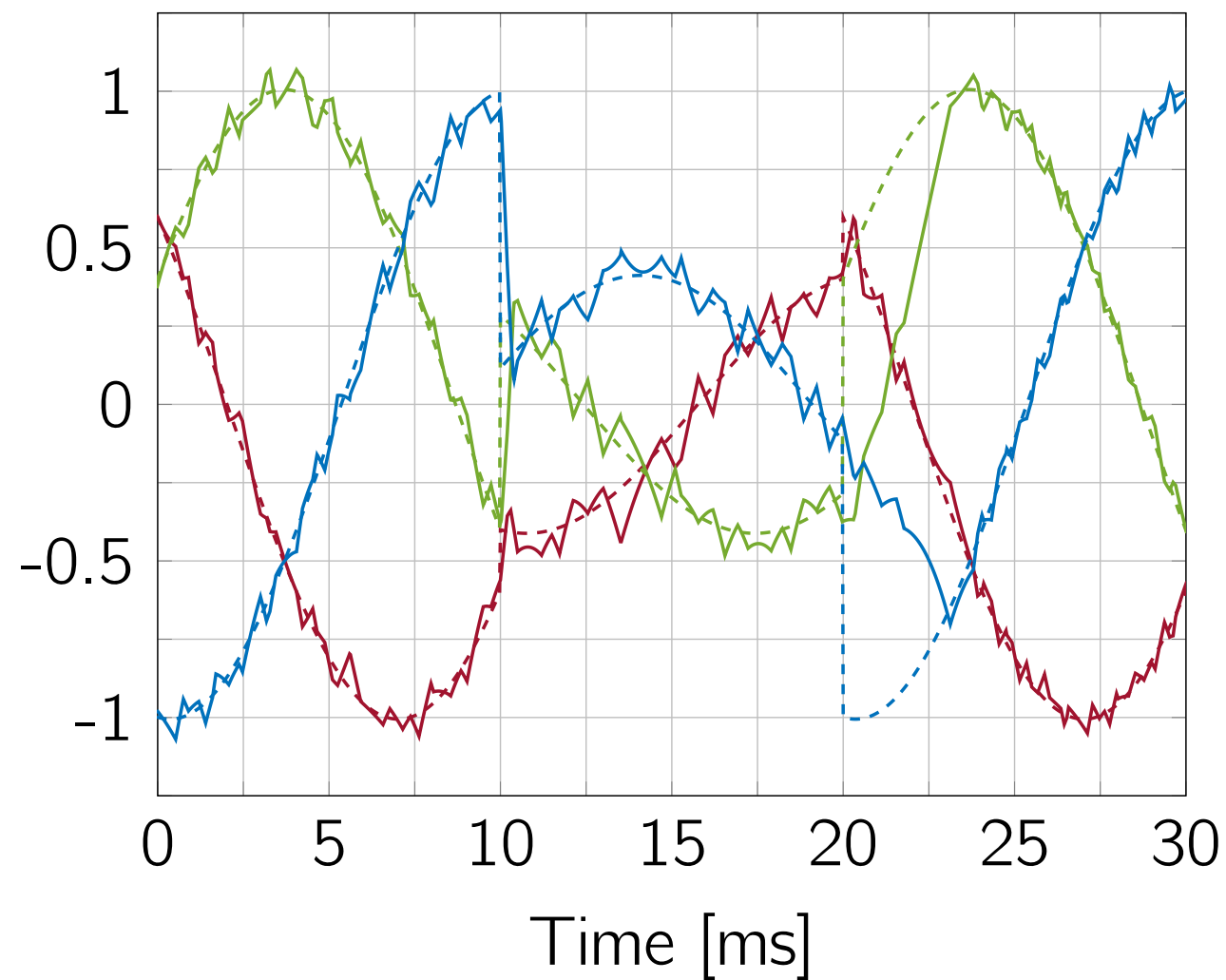
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“High-Speed Finite Control Set Model Predictive Control for Power Electronics”

B. Stellato, T. Geyer, P. J. Goulart

IEEE Transactions on Power Electronics (In Press)

$$T = C(\Psi_r \times i_s)$$



Integer Program

$$\begin{aligned} &\text{minimize} && \frac{1}{2} U^\top Q U + f(x_0)^\top U \\ &\text{subject to} && A_{ineq} U \leq b_{ineq}(x_0) \\ &&& U \in \{-1, 0, 1\}^{3N} \end{aligned}$$

Quadratic Cost Function

$$\int_{\mathcal{X}} V_0(z) c(dz) = \text{Tr}(P_0 \Sigma_c) + 2q_0^\top \mu_c + r_0$$

SDP Formulation

$$\begin{aligned} & \text{maximize} && \text{Tr}(P_0 \Sigma_c) + 2q_0^\top \mu_c + r_0 \\ & \text{subject to} && \tilde{M}_i(m) \succeq 0, \quad \forall m \in \mathcal{M}, \quad i = 1, \dots, M \\ & && V_0 = V_M \\ & && P_i \in \mathbb{S}^{n_x}, \quad q_i \in \mathbb{R}^{n_x}, \quad r_i \in \mathbb{R}, \quad i = 0, \dots, M \end{aligned}$$

